UNSOLVED EXAMPLES

1. A simple Rankine cycle works between pressure of 30 bar and 0.04 bar, the initial condition of steam being dry saturated, calculate the cycle efficiency, work ratio and specific steam consumption.
[**Ans.** 35%, 0.997, 3.84 kg/kWh]
2. A steam power plant works between 40 bar and 0.05 bar. If the steam supplied is dry saturated and the cycle of operation is Rankine, find :
(*i*) Cycle efficiency (*ii*) Specific steam consumption.
[**Ans.** (*i*) 35.5%, (*ii*) 3.8 kg/kWh]
3. Compare the Rankine efficiency of a high pressure plant operating from 80 bar and 400°C and a low pressure plant operating from 40 bar 400°C, if the condenser pressure in both cases is 0.07 bar.
[**Ans.** 0.391 and 0.357]
4. A steam power plant working on Rankine cycle has the range of operation from 40 bar dry saturated to 0.05 bar. Determine :
(*i*) The cycle efficiency (*ii*) Work ratio
(*iii*) Specific fuel consumption. [**Ans.** (*i*) 34.64%, (*ii*) 0.9957, (*iii*) 3.8 kg/kWh]
5. In a Rankine cycle, the steam at inlet to turbine is saturated at a pressure of 30 bar and the exhaust pressure is 0.25 bar. Determine :
(*i*) The pump work (*ii*) Turbine work
(*iii*) Rankine efficiency (*iv*) Condenser heat flow
(*v*) Dryness at the end of expansion.
Assume flow rate of 10 kg/s. [**Ans.** (*i*) 30 kW, (*ii*) 7410 kW, (*iii*) 29.2%, (*iv*) 17900 kW, (*v*) 0.763]
6. In a regenerative cycle the inlet conditions are 40 bar and 400°C. Steam is bled at 10 bar in regenerative heating. The exit pressure is 0.8 bar. Neglecting pump work determine the efficiency of the cycle.
[**Ans.** 0.296]
7. A turbine with one bleeding for regenerative heating of feed water is admitted with steam having enthalpy of 3200 kJ/kg and the exhausted steam has an enthalpy of 2200 kJ/kg. The ideal regenerative feed water heater is fed with 11350 kg/h of bled steam at 3.5 bar (whose enthalpy is 2600 kJ/h). The feed water (condensate from the condenser) with an enthalpy of 134 kJ/kg is pumped to the heater. It leaves the heater dry saturated at 3.5 bar. Determine the power developed by the turbine. [**Ans.** 16015 kW]
8. A binary-vapour cycle operates on mercury and steam. Saturated mercury vapour at 4.5 bar is supplied to the mercury turbine, from which it exhaust at 0.04 bar. The mercury condenser generates saturated steam at 15 bar which is expanded in a steam turbine to 0.04 bar.
(*i*) Find the overall efficiency of the cycle.
(*ii*) If 50000 kg/h of steam flows through the steam turbine, what is the flow through the mercury turbine ?
(*iii*) Assuming that all processes are reversible, what is the useful work done in the binary vapour cycle for the specified steam flow ?
(*iv*) If the steam leaving the mercury condenser is superheated to a temperature of 300°C in a superheater located in the mercury boiler, and if the internal efficiencies of the mercury and steam turbines are 0.85 and 0.87 respectively, calculate the overall efficiency of the cycle. The properties of saturated mercury are given below :

p (bar)	*t (°C)*	h_f (kJ/kg)	h_g	s_f (kJ/kg K)	s_g	v_f (m³/kg)	v_g
4.5	450	63.93	355.98	0.1352	0.5397	79.9×10^{-6}	0.068
0.04	216.9	29.98	329.85	0.0808	0.6925	76.5×10^{-3}	5.178

[**Ans.** (*i*) 52.94%, (*ii*) 59.35×10^4 kg/h, (*iii*) 28.49 MW, (*iv*) 46.2%]

13
Gas Power Cycles

13.1. DEFINITION OF A CYCLE

A **cycle** is defined as a *repeated series of operations occurring in a certain order*. It may be repeated by repeating the processes in the same order. The cycle may be of *imaginary perfect engine or actual engine*. The former is called ***ideal cycle*** and the latter ***actual cycle***. *In ideal cycle all accidental heat losses are prevented and the working substance is assumed to behave like a perfect working substance.*

13.2. AIR STANDARD EFFICIENCY

To compare the effects of different cycles, it is of paramount importance that the effect of the calorific value of the fuel is altogether eliminated and this can be achieved by considering air (which is assumed to behave as a perfect gas) as the working substance in the engine cylinder. *The efficiency of engine using air as the working medium is known as an "**Air standard efficiency**"*. This efficiency is oftenly called ***ideal efficiency***.

The actual efficiency of a cycle is always *less* than the air-standard efficiency of that cycle under ideal conditions. This is taken into account by introducing a new term "***Relative efficiency***" which is defined as :

$$\eta_{relative} = \frac{\text{Actual thermal efficiency}}{\text{Air standard efficiency}} \quad ...(13.1)$$

The analysis of all air standard cycles is based upon the following *assumptions* :

Assumptions :

1. The gas in the engine cylinder is a *perfect gas i.e.*, it obeys the gas laws and has constant specific heats.
2. The physical constants of the gas in the cylinder are the same as those of air at moderate temperatures *i.e.*, the molecular weight of cylinder gas is 29.

$$c_p = 1.005 \text{ kJ/kg-K}, \; c_v = 0.718 \text{ kJ/kg-K}$$

3. The compression and expansion processes are adiabatic and they take place without internal friction, *i.e.*, these processes are *isentropic*.
4. No chemical reaction takes place in the cylinder. Heat is supplied or rejected by bringing a hot body or a cold body in contact with cylinder at appropriate points during the process.

5. The cycle is considered closed with the same 'air' always remaining in the cylinder to repeat the cycle.

13.3. THE CARNOT CYCLE

This cycle has the *highest possible efficiency* and consists of four simple operations namely,

(*a*) Isothermal expansion

(*b*) Adiabatic expansion

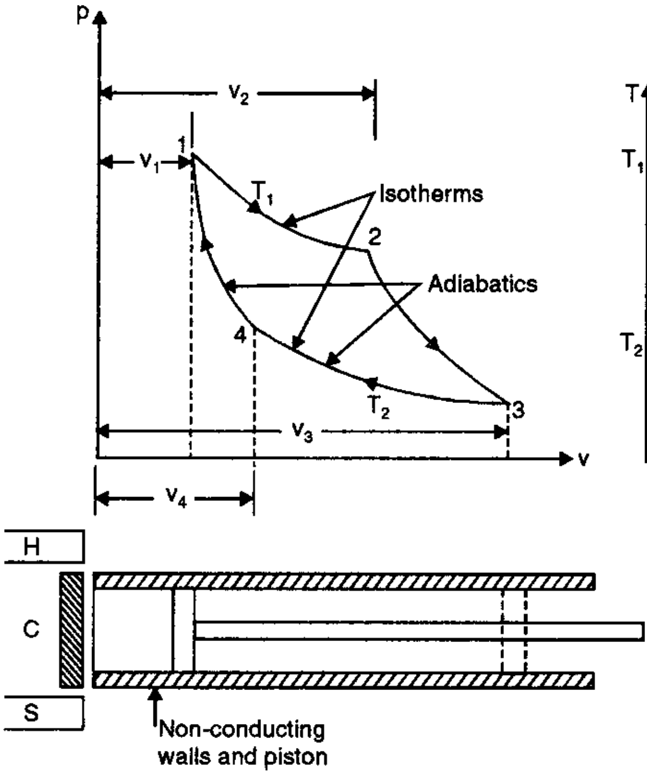
(*c*) Isothermal compression

(*d*) Adiabatic compression.

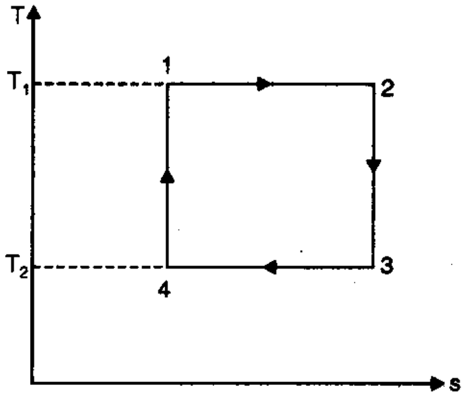
The condition of the Carnot cycle may be imagined to occur in the following way :

One kg of a air is enclosed in the cylinder which (except at the end) is made of perfect non-conducting material. A source of heat '*H*' is supposed to provide unlimited quantity of heat, non-conducting cover '*C*' and a sump '*S*' which is of infinite capacity so that its temperature remains unchanged irrespective of the fact how much heat is supplied to it. The temperature of source *H* is T_1 and the same is of the working substance. The working substance while rejecting heat to sump '*S*' has the temperature. T_2 *i.e.*, the same as that of sump *S*.

Following are the *four stages* of the Carnot cycle. Refer Fig. 13.1 (*a*).



(*a*) Four stages of the carnot cycle



(*b*) *T-s* diagram

Fig. 13.1. Carnot cycle.

Stage (1). Line 1-2 [Fig. 13.1 (*a*)] represents the isothermal expansion which takes place at temperature T_1 when source of heat H is applied to the end of cylinder. Heat supplied in this case is given by $RT_1 \log_e r$ and where r is the ratio of expansion.

Stage (2). Line 2-3 represents the application of non-conducting cover to the end of the cylinder. This is followed by the adiabatic expansion and the temperature falls from T_1 to T_2.

Stage (3). Line 3-4 represents the isothermal compression which takes place when sump 'S' is applied to the end of cylinder. Heat is rejected during this operation whose value is given by $RT_2 \log_e r$ where r is the ratio of compression.

Stage (4). Line 4-1 represents repeated application of non-conducting cover and adiabatic compression due to which temperature increases from T_2 to T_1.

It may be noted that ratio of expansion during isotherm 1-2 and ratio of compression during isotherm 3-4 must be equal to get a closed cycle.

Fig. 13.1 (*b*) represents the Carnot cycle on *T-s* coordinates.

Now according to law of conservation of energy,

Heat supplied = Work done + Heat rejected

Work done = Heat supplied – Heat rejected

$$= RT_1 \,.\, \log_e r - RT_2 \log_e r$$

Efficiency of cycle

$$= \frac{\text{Work done}}{\text{Heat supplied}} = \frac{R \log_e r\,(T_1 - T_2)}{RT_1 \,.\, \log_e r}$$

$$= \frac{T_1 - T_2}{T_1} \qquad ...(13.2)$$

From this equation, it is quite obvious that if temperature T_2 decreases efficiency increases and it becomes 100% if T_2 becomes absolute zero which, of course is impossible to attain. Further more *it is not possible to produce an engine that should work on Carnot's cycle as it would necessitate the piston to travel very slowly during first portion of the forward stroke (isothermal expansion) and to travel more quickly during the remainder of the stroke (adiabatic expansion) which however is not practicable.*

Example 13.1. *A Carnot engine working between 400°C and 40°C produces 130 kJ of work. Determine :*

(*i*) *The engine thermal efficiency.*

(*ii*) *The heat added.*

(*iii*) *The entropy changes during heat rejection process.*

Solution. Temperature, $T_1 = T_2 = 400 + 273 = 673$ K

Temperature, $T_3 = T_4 = 40 + 273 = 313$ K

Work produced, $W = 130$ kJ.

(*i*) **Engine thermal efficiency, η_{th} :**

$$\eta_{th.} = \frac{673 - 313}{673} = \mathbf{0.535 \text{ or } 53.5\%.} \quad \textbf{(Ans.)}$$

(*ii*) **Heat added :**

$$\eta_{th.} = \frac{\text{Work done}}{\text{Heat added}}$$

i.e.,

$$0.535 = \frac{130}{\text{Heat added}}$$

$\therefore$ Heat added

$$= \frac{130}{0.535} = \mathbf{243 \text{ kJ.}} \quad \textbf{(Ans.)}$$

(*iii*) **Entropy change during the heat rejection process, ($S_3 - S_4$) :**

Heat rejected = Heat added – Work done

= 243 – 130 = 113 kJ

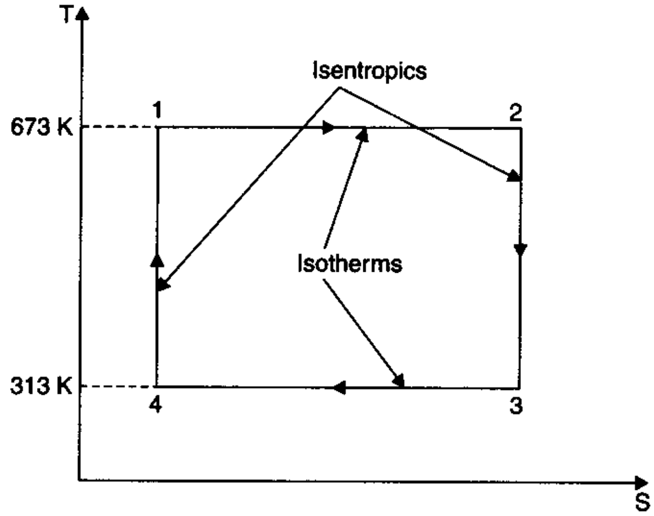


Fig. 13.2

Heat rejected $= T_3 (S_3 - S_4) = 113$

$\therefore \quad (S_3 - S_4) = \frac{113}{T_3} = \frac{113}{313} =$ **0.361 kJ/K. (Ans.)**

Example 13.2. *0.5 kg of air (ideal gas) executes a Carnot power cycle having a thermal efficiency of 50 per cent. The heat transfer to the air during the isothermal expansion is 40 kJ. At the beginning of the isothermal expansion the pressure is 7 bar and the volume is 0.12 m³. Determine :*

(*i*) *The maximum and minimum temperatures for the cycle in K ;*

(*ii*) *The volume at the end of isothermal expansion in m³ ;*

(*iii*) *The heat transfer for each of the four processes in kJ.*

For air $c_v = 0.721$ kJ/kg K, and $c_p = 1.008$ kJ/kg K. **(U.P.S.C. 1993)**

Solution. Refer Fig. 13.3. *Given* : $m = 0.5$ kg ; $\eta_{th} = 50\%$; Heat transferred during isothermal expansion = 40 kJ ; $p_1 = 7$ bar, $V_1 = 0.12$ m³ ; $c_v = 0.721$ kJ/kg K ; $c_p = 1.008$ kJ/kg K.

(*i*) **The maximum and minimum temperatures, T_1, T_2 :**

$$p_1 V_1 = mRT_1$$

$$7 \times 10^5 \times 0.12 = 0.5 \times 287 \times T_1$$

$\therefore$ Maximum temperature, $\mathbf{T_1} = \frac{7 \times 10^5 \times 0.12}{0.5 \times 287} =$ **585.4 K. (Ans.)**

$$\eta_{cycle} = \frac{T_1 - T_2}{T_1} \Rightarrow 0.5 = \frac{585.4 - T_2}{585.4}$$

$\therefore$ Minimum temperature, $\mathbf{T_2} = 585.4 - 0.5 \times 585.4 =$ **292.7 K. (Ans.)**

(*ii*) **The volume at the end of isothermal expansion, V_2 :**

Heat transferred during isothermal expansion

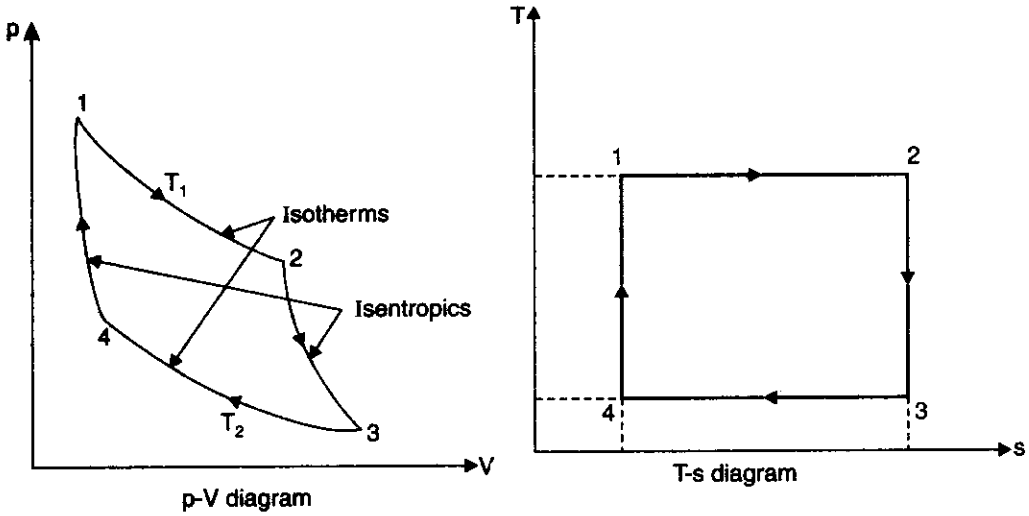


Fig. 13.3. Carnot cycle.

$$= p_1V_1 \ln(r) = mRT_1 \ln\left(\frac{V_2}{V_1}\right) = 40 \times 10^3 \quad \text{...... (Given)}$$

or $$0.5 \times 287 \times 585.4 \ln\left(\frac{V_2}{0.12}\right) = 40 \times 10^3$$

or $$\ln\left(\frac{V_2}{0.12}\right) = \frac{40 \times 10^3}{0.5 \times 287 \times 585.4} = 0.476$$

or $$V_2 = 0.12 \times (e)^{0.476} = \mathbf{0.193\ m^3}. \quad \textbf{(Ans.)}$$

(*iii*) **The heat transfer for each of the four processes :**

Process	*Classification*	*Heat transfer*
1—2	Isothermal expansion	40 kJ
2—3	Adiabatic reversible expansion	zero
3—4	Isothermal compression	– 40 kJ
4—1	Adiabatic reversible compression	zero. **(Ans.)**

☞**Example 13.3.** *In a Carnot cycle, the maximum pressure and temperature are limited to 18 bar and 410°C. The ratio of isentropic compression is 6 and isothermal expansion is 1.5. Assuming the volume of the air at the beginning of isothermal expansion as 0.18 m³, determine :*

(*i*) *The temperature and pressures at main points in the cycle.*

(*ii*) *Change in entropy during isothermal expansion.*

(*iii*) *Mean thermal efficiency of the cycle.*

(*iv*) *Mean effective pressure of the cycle.*

(*v*) *The theoretical power if there are 210 working cycles per minute.*

Solution. Refer Fig. 13.4.

Maximum pressure, $p_1 = 18$ bar

Maximum temperature, $T_1 = (T_2) = 410 + 273 = 683$ K

Ratio of isentropic (or adiabatic) compression, $\frac{V_4}{V_1} = 6$

Ratio of isothermal expansion, $\frac{V_2}{V_1} = 1.5.$

Volume of the air at the beginning of isothermal expansion, $V_1 = 0.18$ m^3.

(*i*) **Temperatures and pressures at the main points in the cycle :**

For the *isentropic process* 4-1 :

$$\frac{T_1}{T_4} = \left(\frac{V_4}{V_1}\right)^{\gamma-1} = (6)^{1.4-1} = (6)^{0.4} = 2.05$$

$$\therefore \quad T_4 = \frac{T_1}{2.05} = \frac{683}{2.05} = 333.2 \text{ K} = T_3$$

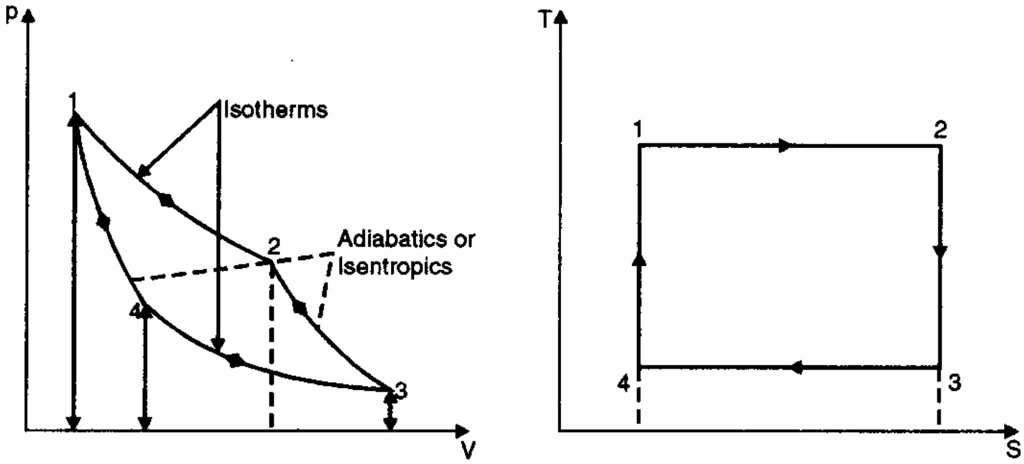


Fig. 13.4

Also, $$\frac{p_1}{p_4} = \left(\frac{V_4}{V_1}\right)^{\gamma} = (6)^{1.4} = 12.29$$

$$\therefore \quad p_4 = \frac{p_1}{12.29} = \frac{18}{12.29} = 1.46 \text{ bar}$$

For the *isothermal process* 1-2 :

$$p_1V_1 = p_2V_2$$

$$p_2 = \frac{p_1V_1}{V_2} = \frac{18}{1.5} = 12 \text{ bar}$$

For *isentropic process* 2-3, we have :

$$p_2V_2^{\gamma} = p_3V_3^{\gamma}$$

$$p_3 = p_2 \times \left(\frac{V_2}{V_3}\right)^{\gamma} = 12 \times \left(\frac{V_1}{V_4}\right)^{\gamma} \qquad \left[\because \frac{V_4}{V_1} = \frac{V_3}{V_2}\right]$$

$$= 12 \times \left(\frac{1}{6}\right)^{1.4} = \mathbf{0.97\ bar.} \quad \textbf{(Ans.)}$$

Hence
p_1 = **18 bar** $T_1 = T_2 = 683$ K
p_2 = **12 bar**
p_3 = **0.97 bar** $T_3 = T_4 = 333.2$ K
p_4 = **1.46 bar** **(Ans.)**

(*ii*) **Change in entropy :**

Change in entropy during isothermal expansion,

$$S_2 - S_1 = mR \log_e \left(\frac{V_2}{V_1}\right) = \frac{p_1V_1}{T_1} \log_e \left(\frac{V_2}{V_1}\right) \qquad \left[\begin{array}{l}\because \ pV = mRT \\ \text{or} \ \ mR = \frac{pV}{T}\end{array}\right]$$

$$= \frac{18 \times 10^5 \times 0.18}{10^3 \times 683} \log_e (1.5) = \mathbf{0.192\ kJ/K.} \quad \textbf{(Ans.)}$$

(*iii*) **Mean thermal efficiency of the cycle :**

Heat supplied, $Q_s = p_1V_1 \log_e \left(\frac{V_2}{V_1}\right)$

$= T_1 (S_2 - S_1)$

$= 683 \times 0.192 = 131.1$ kJ

Heat rejected, $Q_r = p_4V_4 \log_e \left(\frac{V_3}{V_4}\right)$

$= T_4 (S_3 - S_4)$ because increase in entropy during heat addition is equal to decrease in entropy during heat rejection.

$\therefore$ $Q_r = 333.2 \times 0.192 = 63.97$ kJ

$\therefore$ *Efficiency*, $\eta = \frac{Q_s - Q_r}{Q_s} = 1 - \frac{Q_r}{Q_s}$

$$= 1 - \frac{63.97}{131.1} = \mathbf{0.512} \ \ \textbf{or} \ \ \mathbf{51.2\%.} \quad \textbf{(Ans.)}$$

(*iv*) **Mean effective pressure of the cycle, p_m :**

The mean effective pressure of the cycle is given by

$$p_m = \frac{\text{Work done per cycle}}{\text{Stroke volume}}$$

$$\frac{V_3}{V_1} = 6 \times 1.5 = 9$$

Stroke volume, $V_s = V_3 - V_1 = 9V_1 - V_1 = 8V_1 = 8 \times 0.18 = 1.44$ m^3

$\therefore$ $p_m = \frac{(Q_s - Q_r) \times J}{V_s} = \frac{(Q_s - Q_r) \times 1}{V_s}$ $(\because J = 1)$

$$= \frac{(131.1 - 63.97) \times 10^3}{1.44 \times 10^5} = \mathbf{0.466\ bar.} \quad \textbf{(Ans.)}$$

(*v*) **Power of the engine, P :**

Power of the engine working on this cycle is given by

$$P = (131.1 - 63.97) \times (210/60) = \mathbf{234.9\ kW.}\ \ \mathbf{(Ans.)}$$

Example 13.4. *A reversible engine converts one-sixth of the heat input into work. When the temperature of the sink is reduced by 70°C, its efficiency is doubled. Find the temperature of the source and the sink.*

Solution. Let, T_1 = temperature of the source (K), and

T_2 = temperature of the sink (K)

First case :

$$\frac{T_1 - T_2}{T_1} = \frac{1}{6}$$

i.e., $6T_1 - 6T_2 = T_1$

or $5T_1 = 6T_2$ or $T_1 = 1.2T_2$ ...(*i*)

Second case :

$$\frac{T_1 - [T_2 - (70 + 273)]}{T_1} = \frac{1}{3}$$

$$\frac{T_1 - T_2 + 343}{T_1} = \frac{1}{3}$$

$$3T_1 - 3T_2 + 1029 = T_1$$

$$2T_1 = 3T_2 - 1029$$

$$2 \times (1.2T_2) = 3T_2 - 1029 \qquad (\because T_1 = 1.2T_2)$$

$$2.4T_2 = 3T_2 - 1029$$

or $0.6T_2 = 1029$

$\therefore$ $T_2 = \frac{1029}{0.6} = \mathbf{1715\ K}$ **or 1442°C. (Ans.)**

and $T_1 = 1.2 \times 1715 = \mathbf{2058\ K}$ **or 1785°C. (Ans.)**

Example 13.5. *An inventor claims that a new heat cycle will develop 0.4 kW for a heat addition of 32.5 kJ/min. The temperature of heat source is 1990 K and that of sink is 850 K. Is his claim possible ?*

Solution. Temperature of heat source, $T_1 = 1990$ K

Temperature of sink, $T_2 = 850$ K

Heat supplied, = 32.5 kJ/min

Power developed by the engine, $P = 0.4$ kW

The *most efficient engine is one that works on Carnot cycle*

$$\eta_{carnot} = \frac{T_1 - T_2}{T_1} = \frac{1990 - 850}{1990} = 0.573 \text{ or } 57.3\%$$

Also, thermal efficiency of the engine,

$$\eta_{th} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{0.4}{(32.5/60)} = \frac{0.4 \times 60}{32.5}$$

$$= 0.738 \text{ or } 73.8\%$$

which is not feasible as no engine can be more efficient than that working on Carnot cycle.

Hence claims of the inventor is **not true. (Ans.)**

Example 13.6. *An ideal engine operates on the Carnot cycle using a perfects gas as the working fluid. The ratio of the greatest to the least volume is fixed and is x : 1, the lower temperature of the cycle is also fixed, but the volume compression ratio 'r' of the reversible adiabatic compression is variable. The ratio of the specific heats is γ.*

Show that if the work done in the cycle is a maximum then,

$$(\gamma - 1)\log_e \frac{x}{r} + \frac{1}{r^{\gamma-1}} - 1 = 0.$$

Solution. Refer Fig. 13.1.

$$\frac{V_3}{V_1} = x \; ; \; \frac{V_4}{V_1} = r$$

During isotherms, since compression ratio = expansion ratio

$$\therefore \qquad \frac{V_3}{V_4} = \frac{V_2}{V_1}$$

Also
$$\frac{V_3}{V_4} = \frac{V_3}{V_1} \times \frac{V_1}{V_4} = x \times \frac{1}{r} = \frac{x}{r}$$

Work done per kg of the gas

$$= \text{Heat supplied} - \text{Heat rejected} = RT_1 \log_e \frac{x}{r} - RT_2 \log_e \frac{x}{r}$$

$$= R(T_1 - T_2)\log_e \frac{x}{r} = RT_2\left(\frac{T_1}{T_2} - 1\right)\log_e \frac{x}{r}$$

But
$$\frac{T_1}{T_2} = \left(\frac{V_4}{V_1}\right)^{\gamma-1} = (r)^{\gamma-1}$$

$\therefore$ Work done per kg of the gas,

$$W = RT_2\,(r^{\gamma-1} - 1)\log_e \frac{x}{r}$$

Differentiating W w.r.t. 'r' and equating to zero

$$\frac{dW}{dr} = RT_2\left[(r^{\gamma-1} - 1)\left\{\frac{r}{x} \times (-xr^{-2})\right\} + \log_e \frac{x}{r}\{(\gamma - 1)r^{\gamma-2}\}\right] = 0$$

or
$$(r^{\gamma-1} - 1)\left(-\frac{1}{r}\right) + (\gamma - 1) \times r^{\gamma-2}\log_e \frac{x}{r} = 0$$

or
$$-r^{\gamma-2} + \frac{1}{r} + r^{\gamma-2}(\gamma - 1)\log_e \frac{x}{r} = 0$$

or
$$r^{\gamma-2}\left\{-1 + \frac{1}{r \, . \, r^{\gamma-2}} + (\gamma - 1)\log_e \frac{x}{r}\right\} = 0$$

or
$$-1 + \frac{1}{r \, . \, r^{\gamma-2}} + (\gamma - 1)\log_e \frac{x}{r} = 0$$

$$(\gamma - 1)\log_e \frac{x}{r} + \frac{1}{r^{\gamma-1}} - 1 = 0. \qquad \text{...... \textbf{Proved.}}$$

13.4. CONSTANT VOLUME OR OTTO CYCLE

This cycle is so named as it was conceived by 'Otto'. On this cycle, petrol, gas and many types of oil engines work. It is the standard of comparison for internal combustion engines.

Figs. 13.5 (*a*) and (*b*) shows the theoretical *p-V* diagram and *T-s* diagrams of this cycle respectively.

- The point 1 represents that cylinder is full of air with volume V_1, pressure p_1 and absolute temperature T_1.
- Line 1-2 represents the *adiabatic compression* of air due to which p_1, V_1 and T_1 change to p_2, V_2 and T_2, respectively.
- Line 2-3 shows the *supply of heat* to the air *at constant volume* so that p_2 and T_2 change to p_3 and T_3 (V_3 being the same as V_2).
- Line 3-4 represents the *adiabatic expansion* of the air. During expansion p_3, V_3 and T_3 change to a final value of p_4, V_4 or V_1 and T_4, respectively.
- Line 4-1 shows the *rejection of heat* by air at *constant volume* till original state (point 1) reaches.

Consider **1 kg of air** (working substance) :

Heat supplied at constant volume = $c_v(T_3 - T_2)$.

Heat rejected at constant volume = $c_v (T_4 - T_1)$.

But, work done = Heat supplied – Heat rejected

$$= c_v (T_3 - T_2) - c_v (T_4 - T_1)$$

$$\therefore \quad \text{Efficiency} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{c_v (T_3 - T_2) - c_v (T_4 - T_1)}{c_v (T_3 - T_2)}$$

$$= 1 - \frac{T_4 - T_1}{T_3 - T_2} \qquad ...(i)$$

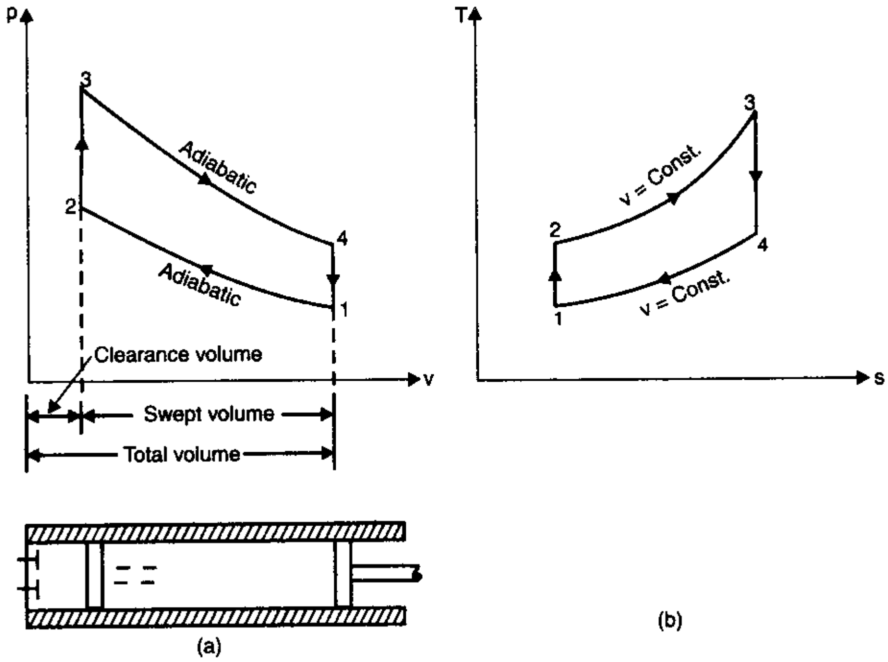


Fig. 13.5

Let compression ratio, $r_c\ (= r) = \frac{v_1}{v_2}$

and expansion ratio, $r_e\ (= r) = \frac{v_4}{v_3}$

(These *two ratios are same* in this cycle)

As $$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1}$$

Then, $$T_2 = T_1 \cdot (r)^{\gamma - 1}$$

Similarly, $$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma - 1}$$

or $$T_3 = T_4 \cdot (r)^{\gamma - 1}$$

Inserting the values of T_2 and T_3 in equation (*i*), we get

$$\eta_{otto} = 1 - \frac{T_4 - T_1}{T_4 \cdot (r)^{\gamma - 1} - T_1 \cdot (r)^{\gamma - 1}} = 1 - \frac{T_4 - T_1}{r^{\gamma - 1}(T_4 - T_1)}$$

$$= 1 - \frac{1}{(r)^{\gamma - 1}} \quad ...(13.3)$$

This expression is known as the ***air standard efficiency of the Otto cycle.***

It is clear from the above expression that efficiency increases with the increase in the value of r, which means we can have maximum efficiency by increasing r to a considerable extent, but *due to practical difficulties its value is limited to about 8.*

The *net work done per kg* in the Otto cycle can also be expressed in terms of p, v. If p is expressed in bar *i.e.*, 10^5 N/m^2, then work done

$$W = \left(\frac{p_3v_3 - p_4v_4}{\gamma - 1} - \frac{p_2v_2 - p_1v_1}{\gamma - 1}\right) \times 10^2 \text{ kJ} \quad ...(13.4)$$

Also $$\frac{p_3}{p_4} = r^{\gamma} = \frac{p_2}{p_1}$$

$\therefore$ $$\frac{p_3}{p_2} = \frac{p_4}{p_1} = r_p$$

where r_p stands for *pressure ratio.*

and $$v_1 = rv_2 = v_4 = rv_3 \qquad \left[\because \frac{v_1}{v_2} = \frac{v_4}{v_3} = r\right]$$

$\therefore$ $$W = \frac{1}{\gamma - 1}\left[p_4v_4\left(\frac{p_3v_3}{p_4v_4} - 1\right) - p_1v_1\left(\frac{p_2v_2}{p_1v_1} - 1\right)\right]$$

$$= \frac{1}{\gamma - 1}\left[p_4v_4\left(\frac{p_3}{p_4r} - 1\right) - p_1v_1\left(\frac{p_2}{p_1r} - 1\right)\right]$$

$$= \frac{v_1}{\gamma - 1}\left[p_4\,(r^{\gamma - 1} - 1) - p_1\,(r^{\gamma - 1} - 1)\right]$$

$$= \frac{v_1}{\gamma - 1}\left[(r^{\gamma - 1} - 1)(p_4 - p_1)\right]$$

$$= \frac{p_1 v_1}{\gamma - 1}\left[(r^{\gamma - 1} - 1)(r_p - 1)\right] \quad ...[13.4\ (a)]$$

Mean effective pressure (p_m) is given by :

$$p_m = \left[\left(\frac{p_3 v_3 - p_4 v_4}{\gamma - 1} - \frac{p_2 v_2 - p_1 v_1}{\gamma - 1}\right) \div (v_1 - v_2)\right] \text{ bar} \quad ...(13.5)$$

Also

$$p_m = \frac{\left[\frac{p_1 v_1}{\gamma - 1}(r^{\gamma - 1} - 1)(r_p - 1)\right]}{(v_1 - v_2)}$$

$$= \frac{\frac{p_1 v_1}{\gamma - 1}[(r^{\gamma - 1} - 1)(r_p - 1)]}{v_1 - \frac{v_1}{r}}$$

$$= \frac{\frac{p_1 v_1}{\gamma - 1}[(r^{\gamma - 1} - 1)(r_p - 1)]}{v_1\left(\frac{r - 1}{r}\right)}$$

i.e.,

$$\mathbf{p_m} = \frac{p_1 r[(r^{\gamma - 1} - 1)(r_p - 1)]}{(\gamma - 1)(r - 1)} \quad ...(13.6)$$

Example 13.7. *The efficiency of an Otto cycle is 60% and $\gamma = 1.5$. What is the compression ratio ?*

Solution. Efficiency of Otto cycle, $\eta = 60\%$

Ratio of specific heats, $\gamma = 1.5$

Compression ratio, $r = ?$

Efficiency of Otto cycle is given by,

$$\eta_{Otto} = 1 - \frac{1}{(r)^{\gamma - 1}}$$

$$0.6 = 1 - \frac{1}{(r)^{1.5 - 1}}$$

or

$$\frac{1}{(r)^{0.5}} = 0.4 \quad \text{or} \quad (r)^{0.5} = \frac{1}{0.4} = 2.5 \quad \text{or} \quad r = 6.25$$

Hence, *compression ratio* = **6.25. (Ans.)**

Example 13.8. *An engine of 250 mm bore and 375 mm stroke works on Otto cycle. The clearance volume is 0.00263 m³. The initial pressure and temperature are 1 bar and 50°C. If the maximum pressure is limited to 25 bar, find the following :*

(i) The air standard efficiency of the cycle.

(ii) The mean effective pressure for the cycle.

Assume the ideal conditions.

Solution. Bore of the engine, $D = 250 \text{ mm} = 0.25 \text{ m}$
Stroke of the engine, $L = 375 \text{ mm} = 0.375 \text{ m}$
Clearance volume, $V_c = 0.00263 \text{ m}^3$
Initial pressure, $p_1 = 1 \text{ bar}$
Initial temperature, $T_1 = 50 + 273 = 323 \text{ K}$

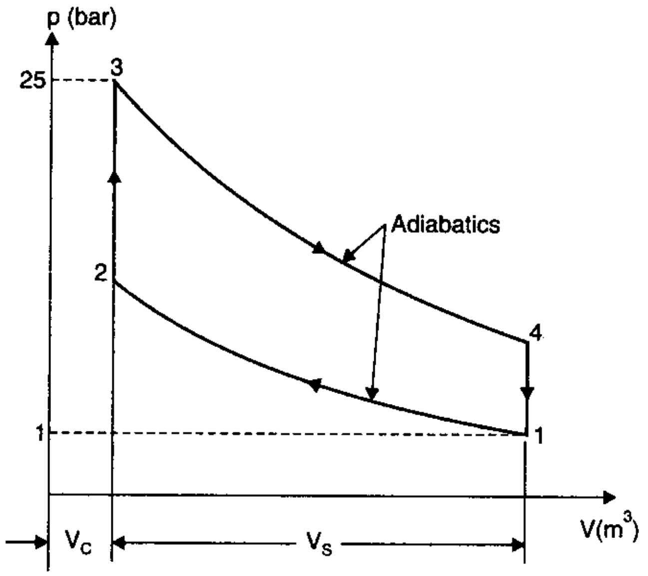


Fig. 13.6

Maximum pressure, $p_3 = 25 \text{ bar}$
Swept volume, $V_s = \pi/4\ D^2L = \pi/4 \times 0.25^2 \times 0.375 = 0.0184 \text{ m}^3$
Compression ratio, $r = \dfrac{V_s + V_c}{V_c} = \dfrac{0.0184 + 0.00263}{0.00263} = 8.$

(*i*) **Air standard efficiency :**

The air standard efficiency of Otto cycle is given by

$$\eta_{Otto} = 1 - \frac{1}{(r)^{\gamma - 1}} = 1 - \frac{1}{(8)^{1.4 - 1}} = 1 - \frac{1}{(8)^{0.4}}$$

$$= 1 - 0.435 = \mathbf{0.565} \textbf{ or } \mathbf{56.5\%.} \textbf{ (Ans.)}$$

(*ii*) **Mean effective pressure, p_m :**

For adiabatic (or isentropic) process 1-2

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

or $$p_2 = p_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 1 \times (r)^{1.4} = 1 \times (8)^{1.4} = 18.38 \text{ bar}$$

∴ Pressure ratio, $r_p = \dfrac{p_3}{p_2} = \dfrac{25}{18.38} = 1.36$

The mean effective pressure is given by

$$p_m = \frac{p_1r[(r^{\gamma-1} - 1)(r_p - 1)]}{(\gamma - 1)(r - 1)} = \frac{1 \times 8[\{(8)^{1.4-1} - 1\}(1.36 - 1)]}{(1.4 - 1)(8 - 1)}$$

... [Eqn. (13.6)]

$$= \frac{8(2.297-1)(0.36)}{0.4 \times 7} = 1.334 \text{ bar}$$

Hence *mean effective pressure* = **1.334 bar. (Ans.)**

Example 13.9. *The minimum pressure and temperature in an Otto cycle are 100 kPa and 27°C. The amount of heat added to the air per cycle is 1500 kJ/kg.*

(i) Determine the pressures and temperatures at all points of the air standard Otto cycle.

(ii) Also calculate the specific work and thermal efficiency of the cycle for a compression ratio of 8 : 1.

Take for air : $c_v = 0.72$ *kJ/kg K, and* $\gamma = 1.4$. **(GATE, 1998)**

Solution. Refer Fig. 13.7. *Given :* p_1 = 100 kPa = 10^5 N/m² or 1 bar ;

$T_1 = 27 + 273 = 300$ K ; Heat added = 1500 kJ/kg ;

$r = 8 : 1$; $c_v = 0.72$ kJ/kg ; $\gamma = 1.4$.

Consider 1 kg of air.

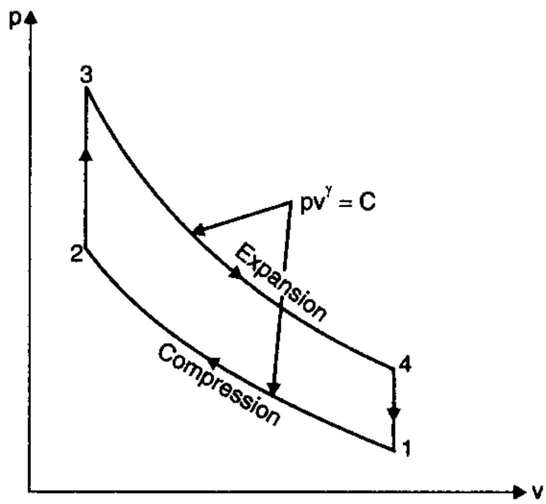


Fig. 13.7

(*i*) **Pressures and temperatures at all points :**

Adiabatic compression process 1-2 :

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (r)^{\gamma-1} = (8)^{1.4-1} = 2.297$$

$\therefore$ $\mathbf{T_2} = 300 \times 2.297 = $ **689.1 K. (Ans.)**

Also $p_1 v_1^{\gamma} = p_2 v_2^{\gamma}$

or $$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = (8)^{1.4} = 18.379$$

$\therefore$ $\mathbf{p_2} = 1 \times 18.379 = $ **18.379 bar. (Ans.)**

Constant volume process 2-3 :

Heat added during the process,

$$c_v (T_3 - T_2) = 1500$$

or $$0.72\ (T_3 - 689.1) = 1500$$

or $$\mathbf{T_3} = \frac{1500}{0.72} + 689.1 = \mathbf{2772.4\ K.}\ \textbf{(Ans.)}$$

Also, $$\frac{p_2}{T_2} = \frac{p_3}{T_3} \Rightarrow \mathbf{p_3} = \frac{p_2 T_3}{T_2} = \frac{18.379 \times 2772.4}{689.1} = \mathbf{73.94\ bar.}\ \textbf{(Ans.)}$$

Adiabatic Expansion process 3-4 :

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma - 1} = (r)^{\gamma - 1} = (8)^{1.4 - 1} = 2.297$$

$\therefore$ $$\mathbf{T_4} = \frac{T_3}{2.297} = \frac{2772.4}{2.297} = \mathbf{1206.9\ K.}\ \textbf{(Ans.)}$$

Also, $$p_3 v_3^{\gamma} = p_4 v_4^{\gamma} \Rightarrow \mathbf{p_4} = p_3 \times \left(\frac{v_3}{v_4}\right)^{\gamma} = 73.94 \times \left(\frac{1}{8}\right)^{1.4} = \mathbf{4.023\ bar.}\ \textbf{(Ans.)}$$

(*ii*) **Specific work and thermal efficiency :**

Specific work = Heat added – heat rejected

$$= c_v (T_3 - T_2) - c_v (T_4 - T_1) = c_v [(T_3 - T_2) - (T_4 - T_1)]$$
$$= 0.72 [(2772.4 - 689.1) - (1206.9 - 300)] = \mathbf{847\ kJ/kg.}\ \textbf{(Ans.)}$$

Thermal efficiency, $$\eta_{th} = 1 - \frac{1}{(r)^{\gamma - 1}}$$

$$= 1 - \frac{1}{(8)^{1.4 - 1}} = \mathbf{0.5647}\ \text{ or }\ \mathbf{56.47\%.}\ \textbf{(Ans.)}$$

Example 13.10. *An air standard Otto cycle has a volumetric compression ratio of 6, the lowest cycle pressure of 0.1 MPa and operates between temperature limits of 27°C and 1569°C.*

(i) Calculate the temperature and pressure after the isentropic expansion (ratio of specific heats = 1.4).

(ii) Since it is observed that values in (i) are well above the lowest cycle operating conditions, the expansion process was allowed to continue down to a pressure of 0.1 MPa. Which process is required to complete the cycle ? Name the cycle so obtained.

(iii) Determine by what percentage the cycle efficiency has been improved. **(GATE, 1994)**

Solution. Refer Fig. 13.8. *Given :* $\frac{v_1}{v_2} = \frac{v_4}{v_3} = r = 6$; $p_1 = 0.1$ MPa = 1 bar ; $T_1 = 27 + 273 = 300$ K ; $T_3 = 1569 + 273 = 1842$ K ; $\gamma = 1.4$.

(*i*) **Temperature and pressure after the isentropic expansion, T_4, p_4 :**

Consider 1 kg of air :

For *the compression process 1-2 :*

$$p_1 v_1^{\gamma} = p_2 v_2^{\gamma} \Rightarrow p_2 = p_1 \times \left(\frac{v_1}{v_2}\right)^{\gamma} = 1 \times (6)^{1.4} = 12.3 \text{ bar}$$

Also $$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = (6)^{1.4 - 1} = 2.048$$

$\therefore$ $$T_2 = 300 \times 2.048 = 614.4 \text{ K}$$

For *the constant volume process 2-3 :*

$$\frac{p_2}{T_2} = \frac{p_3}{T_3} \Rightarrow p_3 = \frac{p_2 T_3}{T_2} = 12.3 \times \frac{1842}{614.4} = 36.9 \text{ bar}$$

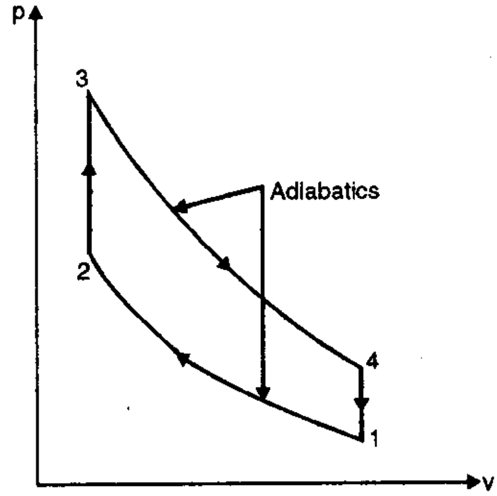


Fig. 13.8

For *the expansion process 3-4 :*

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma - 1} = (6)^{1.4 - 1} = 2.048$$

$$\therefore \quad \mathbf{T_4} = \frac{T_3}{2.048} = \frac{1842}{2.048} = \mathbf{900\ K.} \quad \textbf{(Ans.)}$$

Also $$p_3 v_3^{\gamma} = p_4 v_4^{\gamma} \Rightarrow p_4 = p_3 \times \left(\frac{v_3}{v_4}\right)^{\gamma}$$

or $$\mathbf{p_4} = 36.9 \times \left(\frac{1}{6}\right)^{1.4} = \mathbf{3\ bar.} \quad \textbf{(Ans.)}$$

(*ii*) **Process required to complete the cycle :**

Process required to complete the cycle is the *constant pressure scavenging.*

The cycle is called **Atkinson cycle** (Refer Fig. 13.9).

(*iii*) **Percentage improvement/increase in efficiency :**

$$\eta_{Otto} = 1 - \frac{1}{(r)^{\gamma - 1}} = 1 - \frac{1}{(6)^{1.4 - 1}} = 0.5116 \text{ or } \mathbf{51.16\%.} \quad \textbf{(Ans.)}$$

$$\eta_{Atkinson} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}}$$

$$= \frac{c_v(T_3 - T_2) - c_p(T_5 - T_1)}{c_v(T_3 - T_2)} = 1 - \frac{c_p(T_5 - T_1)}{c_v(T_3 - T_2)} = 1 - \frac{\gamma(T_5 - T_1)}{(T_3 - T_2)}$$

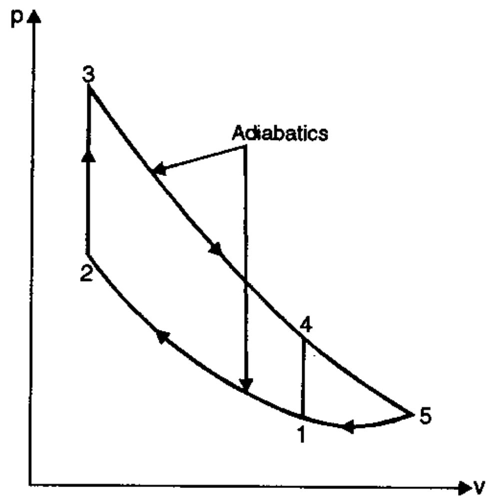


Fig. 13.9. Atkinson cycle.

Now, $$\frac{T_5}{T_3} = \left(\frac{p_5}{p_3}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{or} \quad T_5 = 1842 \times \left(\frac{1.0}{36.9}\right)^{\frac{1.4-1}{1.4}} = 657 \text{ K}$$

$\therefore$ $$\eta_{Atkinson} = 1 - \frac{1.4(657 - 300)}{(1842 - 614.4)} = \mathbf{0.5929 \text{ or } 59.29\%.}$$

$\therefore$ **Improvement in efficiency** = 59.29 – 51.16 = **8.13%. (Ans.)**

Example 13.11. *A certain quantity of air at a pressure of 1 bar and temperature of 70°C is compressed adiabatically until the pressure is 7 bar in Otto cycle engine. 465 kJ of heat per kg of air is now added at constant volume. Determine :*

(*i*) *Compression ratio of the engine.*

(*ii*) *Temperature at the end of compression.*

(*iii*) *Temperature at the end of heat addition.*

Take for air c_p = *1.0 kJ/kg K,* c_v = *0.706 kJ/kg K.*

Show each operation on p-V and T-s diagrams.

Solution. Refer Fig. 13.10.

Initial pressure, p_1 = 1 bar

Initial temperature, T_1 = 70 + 273 = 343 K

Pressure after adiabatic compression, p_2 = 7 bar

Heat addition at constant volume, Q_s = 465 kJ/kg of air

Specific heat at constant pressure, c_p = 1.0 kJ/kg K

Specific heat at constant volume, c_v = 0.706 kJ/kg K

$\therefore$ $$\gamma = \frac{c_p}{c_v} = \frac{1.0}{0.706} = 1.41$$

(*i*) **Compression ratio of engine, r :**

According to *adiabatic compression 1-2*

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$

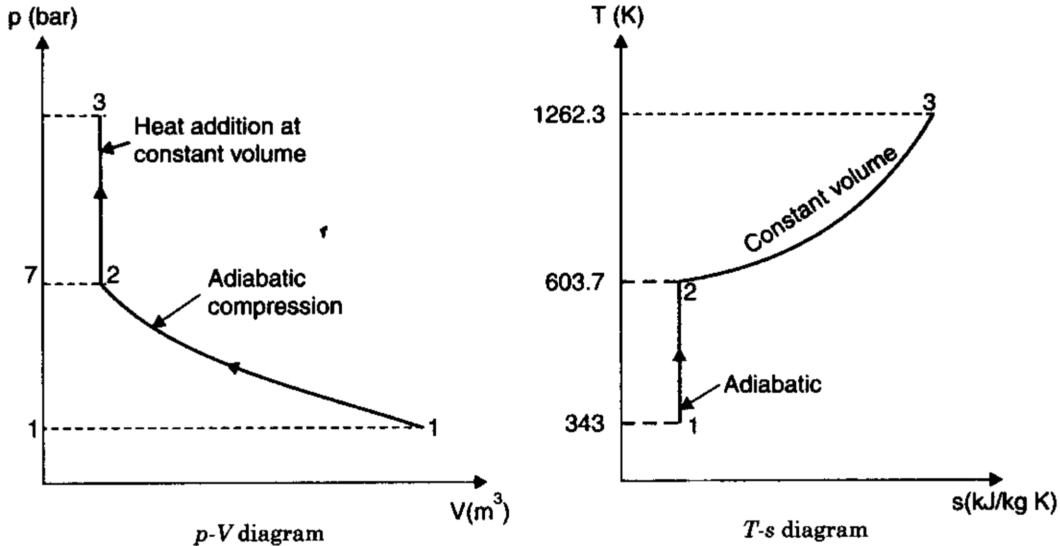


Fig. 13.10

or $$\left(\frac{V_1}{V_2}\right)^{\gamma} = \frac{p_2}{p_1}$$

or $$(r)^{\gamma} = \frac{p_2}{p_1} \qquad \left(\because \frac{V_1}{V_2} = r\right)$$

or $$r = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \left(\frac{7}{1}\right)^{\frac{1}{1.41}} = (7)^{0.709} = 3.97$$

Hence *compression ratio of the engine* = **3.97. (Ans.)**

(*ii*) **Temperature at the end of compression, T_2 :**

In case of *adiabatic compression 1-2,*

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = (3.97)^{1.41-1} = 1.76$$

$$\therefore \qquad T_2 = 1.76\ T_1 = 1.76 \times 343 = 603.7 \text{ K or } 330.7°\text{C}$$

Hence *temperature at the end of compression* = **330.7°C. (Ans.)**

(*iii*) **Temperature at the end of heat addition, T_3 :**

According to *constant volume heating operation 2-3*

$$Q_s = c_v (T_3 - T_2) = 465$$

$$0.706 (T_3 - 603.7) = 465$$

or $$T_3 - 603.7 = \frac{465}{0.706}$$

or $$T_3 = \frac{465}{0.706} + 603.7 = 1262.3 \text{ K or } 989.3°\text{C}$$

Hence *temperature at the end of heat addition* = **989.3°C. (Ans.)**

Example 13.12. *In a constant volume 'Otto cycle', the pressure at the end of compression is 15 times that at the start, the temperature of air at the beginning of compression is 38°C and maximum temperature attained in the cycle is 1950°C. Determine :*

(*i*) *Compression ratio.*

(*ii*) *Thermal efficiency of the cycle.*

(*iii*) *Work done.*

Take γ for air = 1.4.

Solution. Refer Fig. 13.11.

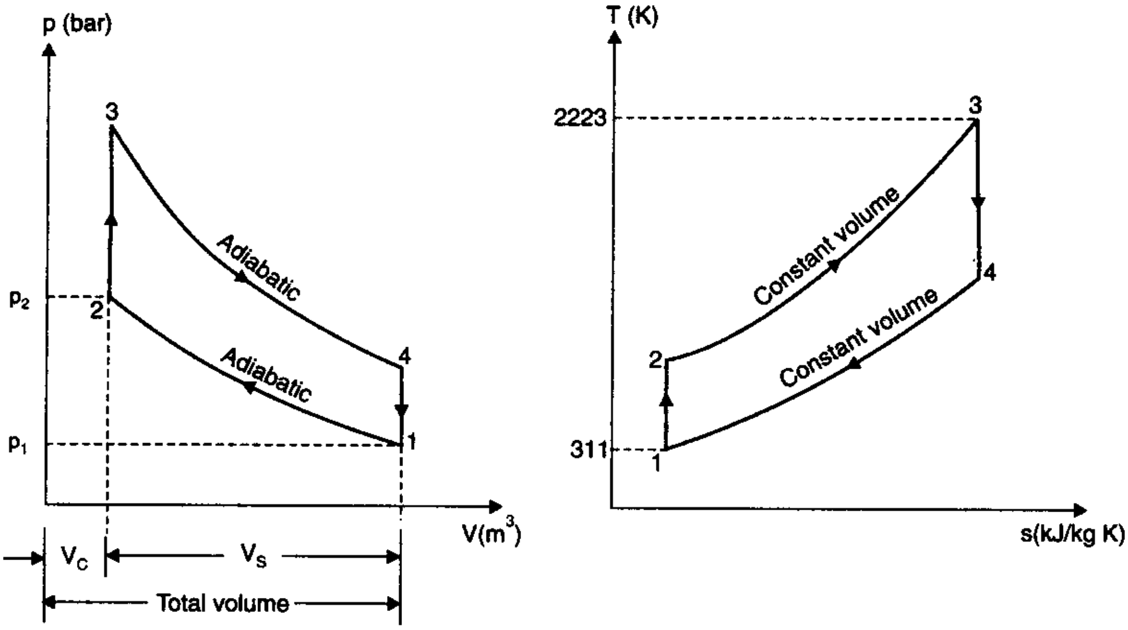


Fig. 13.11

Initial temperature, $T_1 = 38 + 273 = 311$ K

Maximum temperature, $T_3 = 1950 + 273 = 2223$ K.

(*i*) **Compression ratio, r :**

For *adiabatic compression 1-2,*

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

or $$\left(\frac{V_1}{V_2}\right)^{\gamma} = \frac{p_2}{p_1}$$

But $$\frac{p_2}{p_1} = 15 \quad \text{...(given)}$$

∴ $$(r)^{\gamma} = 15 \quad \left[\because\ r = \frac{V_1}{V_2}\right]$$

or $$(r)^{1.4} = 15$$

or $$r = (15)^{\frac{1}{1.4}} = (15)^{0.714} = 6.9$$

Hence *compression ratio* **= 6.9. (Ans.)**

(*ii*) **Thermal efficiency :**

Thermal efficiency, $\eta_{th} = 1 - \dfrac{1}{(r)^{\gamma-1}} = 1 - \dfrac{1}{(6.9)^{1.4-1}} =$ **0.538 or 53.8%. (Ans.)**

(*iii*) **Work done :**

Again, for *adiabatic compression 1-2,*

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r)^{\gamma-1} = (6.9)^{1.4-1} = (6.9)^{0.4} = 2.16$$

or $$T_2 = T_1 \times 2.16 = 311 \times 2.16 = 671.7 \text{ K or } 398.7°\text{C}$$

For *adiabatic expansion process 3-4*

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = (r)^{\gamma-1} = (6.9)^{0.4} = 2.16$$

or $$T_4 = \frac{T_3}{2.16} = \frac{2223}{2.16} = 1029 \text{ K or } 756°\text{C}$$

Heat supplied *per kg of air*

$= c_v(T_3 - T_2) = 0.717(2223 - 671.7)$

$= 1112.3$ kJ/kg or air

$$\left[c_v = \frac{R}{\gamma - 1} = \frac{0.287}{1.4 - 1} = 0.717 \text{ kJ/ kg K}\right]$$

Heat rejected per kg of air

$= c_v(T_4 - T_1) = 0.717(1029 - 311)$

$= 514.8$ kJ/kg of air

∴ *Work done per kg of air* = Heat supplied – heat rejected

$= 1112.3 - 514.8$

= **597.5 kJ or 597500 N-m. (Ans.)**

Example 13.13. *An engine working on Otto cycle has a volume of 0.45 m³, pressure 1 bar and temperature 30°C at the beginning of compression stroke. At the end of compression stroke, the pressure is 11 bar. 210 kJ of heat is added at constant volume. Determine :*

(*i*) *Pressures, temperatures and volumes at salient points in the cycle.*

(*ii*) *Percentage clearance.*

(*iii*) *Efficiency.*

(*iv*) *Net work per cycle.*

(*v*) *Mean effective pressure.*

(*vi*) *Ideal power developed by the engine if the number of working cycles per minute is 210.*

Assume the cycle is reversible.

Solution. Refer Fig. 13.12

Volume, $V_1 = 0.45 \text{ m}^3$

Initial pressure, $p_1 = 1$ bar

Initial temperature, $T_1 = 30 + 273 = 303$ K

Pressure at the end of compression stroke, $p_2 = 11$ bar

Heat added at constant volume = 210 kJ

Number of working cycles/min. = 210.

(*i*) **Pressures, temperatures and volumes at salient points :**

For *adiabatic compression 1-2*

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

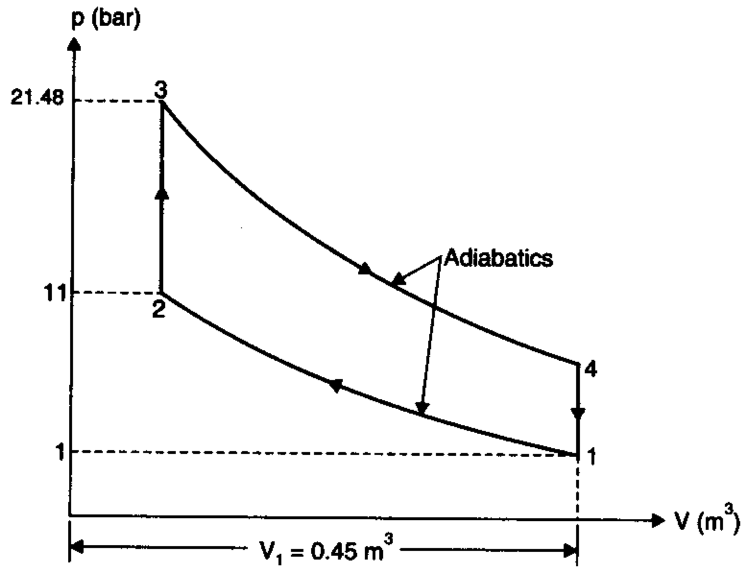


Fig. 13.12

or $$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = (r)^{\gamma} \quad \text{or} \quad r = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \left(\frac{11}{1}\right)^{\frac{1}{1.4}} = (11)^{0.714} = 5.5$$

Also $$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r)^{\gamma-1} = (5.5)^{1.4-1} = 1.977 \simeq 1.98$$

$\therefore$ $$T_2 = T_1 \times 1.98 = 303 \times 1.98 = \mathbf{600\ K.}\ \mathbf{(Ans.)}$$

Applying gas laws to points 1 and 2

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$\therefore$ $$V_2 = \frac{T_2}{T_1} \times \frac{p_1}{p_2} \times V_1 = \frac{600 \times 1 \times 0.45}{303 \times 11} = \mathbf{0.081\ m^3.}\ \mathbf{(Ans.)}$$

The heat supplied during the *process 2-3* is given by :

$$Q_s = m\, c_v\, (T_3 - T_2)$$

where $$m = \frac{p_1V_1}{RT_1} = \frac{1 \times 10^5 \times 0.45}{287 \times 303} = 0.517 \text{ kg}$$

$\therefore$ $$210 = 0.517 \times 0.71\, (T_3 - 600)$$

or $$T_3 = \frac{210}{0.517 \times 0.71} + 600 = \mathbf{1172\ K.}\ \mathbf{(Ans.)}$$

For the *constant volume process 2-3*

$$\frac{p_3}{T_3} = \frac{p_2}{T_2}$$

$\therefore$ $$p_3 = \frac{T_3}{T_2} \times p_2 = \frac{1172}{600} \times 11 = \mathbf{21.48\ bar.}\ \mathbf{(Ans.)}$$

$$V_3 = V_2 = \mathbf{0.081\ m^3.}\ \mathbf{(Ans.)}$$

For the *adiabatic (or isentropic) process 3-4*

$$p_3V_3^\gamma = p_4V_4^\gamma$$

$$p_4 = p_3 \times \left(\frac{V_3}{V_4}\right)^\gamma = p_3 \times \left(\frac{1}{r}\right)^\gamma$$

$$= 21.48 \times \left(\frac{1}{5.5}\right)^{1.4} = \textbf{1.97 bar.} \quad \textbf{(Ans.)}$$

Also
$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \left(\frac{1}{r}\right)^{\gamma-1} = \left(\frac{1}{5.5}\right)^{1.4-1} = 0.505$$

$\therefore$
$$T_4 = 0.505\ T_3 = 0.505 \times 1172 = \textbf{591.8 K.} \quad \textbf{(Ans.)}$$

$$V_4 = V_1 = \textbf{0.45 m}^3. \quad \textbf{(Ans.)}$$

(*ii*) **Percentage clearance :**

Percentage clearance

$$= \frac{V_c}{V_s} = \frac{V_2}{V_1 - V_2} \times 100 = \frac{0.081}{0.45 - 0.081} \times 100$$

$$= \textbf{21.95\%.} \quad \textbf{(Ans.)}$$

(*iii*) **Efficiency :**

The heat rejected per cycle is given by

$$Q_r = mc_v(T_4 - T_1)$$

$$= 0.517 \times 0.71\ (591.8 - 303) = 106 \text{ kJ}$$

The air-standard efficiency of the cycle is given by

$$\eta_{otto} = \frac{Q_s - Q_r}{Q_s} = \frac{210 - 106}{210} = \textbf{0.495 or 49.5\%.} \quad \textbf{(Ans.)}$$

[Alternatively :

$$\eta_{otto} = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(5.5)^{1.4-1}} = \textbf{0.495 or 49.5\%.} \quad \textbf{(Ans.)}$$]

(*iv*) **Mean effective pressure, p_m :**

The mean effective pressure is given by

$$p_m = \frac{W \text{ (work done)}}{V_s \text{(swept volume)}} = \frac{Q_s - Q_r}{(V_1 - V_2)}$$

$$= \frac{(210 - 106) \times 10^3}{(0.45 - 0.081) \times 10^5} = \textbf{2.818 bar.} \quad \textbf{(Ans.)}$$

(*v*) **Power developed, P :**

Power developed, P = Work done per second

= Work done per cycle × number of cycles per second

= (210 − 106) × (210/60) = **364 kW.** **(Ans.)**

Example 13.14. (*a*) *Show that the compression ratio for the maximum work to be done per kg of air in an Otto cycle between upper and lower limits of absolute temperatures T_3 and T_1 is given by*

$$r = \left(\frac{T_3}{T_1}\right)^{1/2(\gamma-1)}$$

(b) Determine the air-standard efficiency of the cycle when the cycle develops maximum work with the temperature limits of 310 K and 1220 K and working fluid is air. What will be the percentage change in efficiency if helium is used as working fluid instead of air ? The cycle operates between the same temperature limits for maximum work development.

Consider that all conditions are ideal.

Solution. Refer Fig. 13.13.

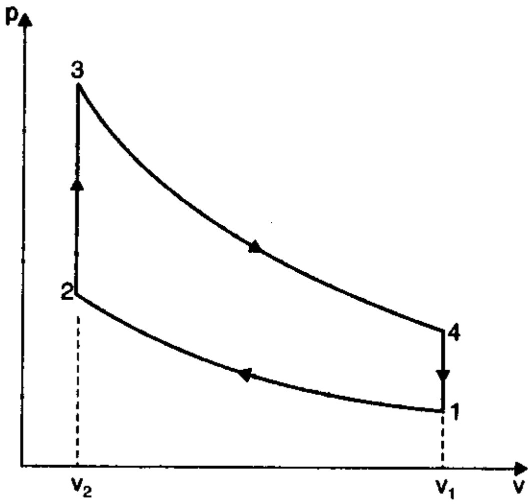


Fig. 13.13

(*a*) The work done per kg of fluid in the cycle is given by

$$W = Q_s - Q_r = c_v (T_3 - T_2) - c_v (T_4 - T_1)$$

But $$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = (r)^{\gamma - 1}$$

$\therefore$ $$T_2 = T_1 \, . \, (r)^{\gamma - 1} \quad ...(i)$$

Similarly, $$T_3 = T_4 \, . \, (r)^{\gamma - 1} \quad ...(ii)$$

$\therefore$ $$W = c_v \left[T_3 - T_1 \, . (r)^{\gamma - 1} - \frac{T_3}{(r)^{\gamma - 1}} + T_1\right] \quad ...(iii)$$

This expression is a function of r when T_3 and T_1 are fixed. The value of W will be maximum when,

$$\frac{dW}{dr} = 0.$$

$\therefore$ $$\frac{dW}{dr} = - T_1 \, . \, (\gamma - 1) \, (r)^{\gamma - 2} - T_3 \, (1 - \gamma) \, (r)^{-\gamma} = 0$$

or $$T_3 \, (r)^{-\gamma} = T_1 \, (r)^{\gamma - 2}$$

or $$\frac{T_3}{T_1} = (r)^{2(\gamma - 1)}$$

$\therefore$ $$r = \left(\frac{T_3}{T_1}\right)^{1/2(\gamma - 1)}$$ **......Proved.**

(*b*) **Change in efficiency :**

For *air* $\gamma = 1.4$

$$\therefore \qquad r = \left(\frac{T_3}{T_1}\right)^{1/2(1.4-1)} = \left(\frac{1220}{310}\right)^{1/0.8} = 5.54$$

The air-standard efficiency is given by

$$\eta_{otto} = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(5.54)^{1.4-1}} = \mathbf{0.495 \text{ or } 49.5\%.} \quad \textbf{(Ans.)}$$

If *helium* is used, then the values of

$$c_p = 5.22 \text{ kJ/kg K and } c_v = 3.13 \text{ kJ/kg K}$$

$$\therefore \qquad \gamma = \frac{c_p}{c_v} = \frac{5.22}{3.13} = 1.67$$

The compression ratio for maximum work for the temperature limits T_1 and T_3 is given by

$$r = \left(\frac{T_3}{T_1}\right)^{1/2(\gamma-1)} = \left(\frac{1220}{310}\right)^{1/2(1.67-1)} = 2.77$$

The air-standard efficiency is given by

$$\eta_{otto} = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(2.77)^{1.67-1}} = \mathbf{0.495 \quad or \quad 49.5\%.}$$

Hence change in efficiency is nil. (Ans.)

Example 13.15. (*a*) *An engine working on Otto cycle, in which the salient points are 1, 2, 3 and 4, has upper and lower temperature limits T_3 and T_1. If the maximum work per kg of air is to be done, show that the intermediate temperature is given by*

$$T_2 = T_4 = \sqrt{T_1 T_3}\,.$$

(*b*) *If an engine works on Otto cycle between temperature limits 1450 K and 310 K, find the maximum power developed by the engine assuming the circulation of air per minute as 0.38 kg.*

Solution. (*a*) Refer Fig. 13.13 (Example 13.14).

Using the equation (*iii*) of example 13.14.

$$W = c_v \left[T_3 - T_1 . (r)^{\gamma-1} - \frac{T_3}{(r)^{\gamma-1}} + T_1\right]$$

and differentiating W w.r.t. r and equating to zero

$$r = \left(\frac{T_3}{T_1}\right)^{1/2(\gamma-1)}$$

$$T_2 = T_1 (r)^{\gamma-1} \text{ and } T_4 = T_3/(r)^{\gamma-1}$$

Substituting the value of r in the above equation, we have

$$T_2 = T_1 \left[\left(\frac{T_3}{T_1}\right)^{1/2(\gamma-1)}\right]^{\gamma-1} = T_1\left(\frac{T_3}{T_1}\right)^{1/2} = \sqrt{T_1 T_3}$$

Similarly,

$$T_4 = \frac{T_3}{\left[\left(\frac{T_3}{T_1}\right)^{1/2(\gamma-1)}\right]^{\gamma-1}} = \frac{T_3}{\left(\frac{T_3}{T_1}\right)^{1/2}} = \sqrt{T_3 T_1}$$

$\therefore$ $T_2 = T_4 = \sqrt{T_1 T_3}$. **Proved.**

(b) **Power developed, P :**

$$\left.\begin{aligned} T_1 &= 310\text{ K} \\ T_3 &= 1450\text{ K} \\ m &= 0.38\text{ kg} \end{aligned}\right\} \quad \text{....(given)}$$

Work done $W = c_v\,[(T_3 - T_2) - (T_4 - T_1)]$

$$T_2 = T_4 = \sqrt{T_1 T_3} = \sqrt{310 \times 1450} = 670.4\text{ K}$$

$\therefore$ $W = 0.71\,[(1450 - 670.4) - (670.4 - 310)]$

$= 0.71\,(779.6 - 360.4) = 297.6$ kJ/kg

Work done per second $= 297.6 \times (0.38/60) = 1.88$ kJ/s

Hence **power developed, P = 1.88 kW. (Ans.)**

Example 13.16. *For the same compression ratio, show that the efficiency of Otto cycle is greater than that of Diesel cycle.*

Solution. Refer Fig. 13.14.

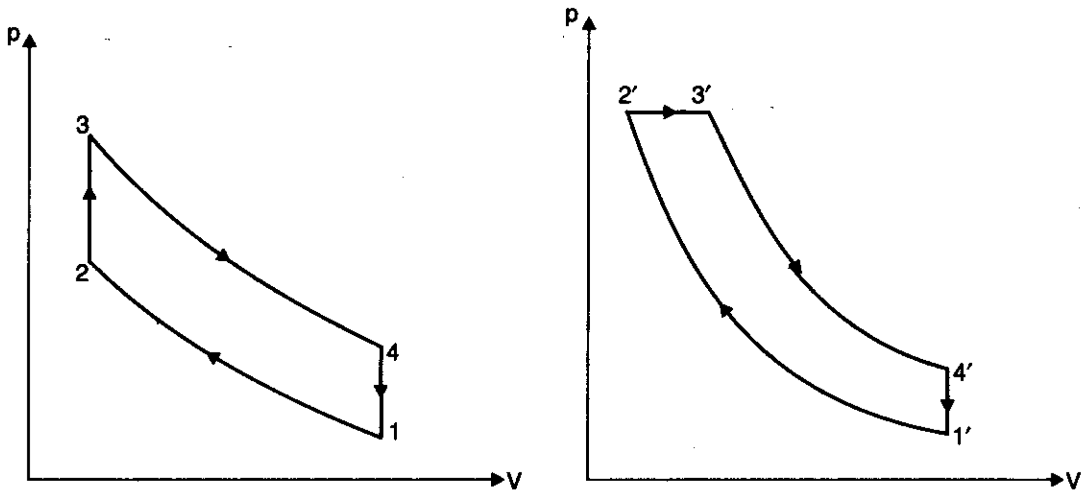


Fig. 13.14

We know that

$$\eta_{\text{Otto}} = 1 - \frac{1}{(r)^{\gamma - 1}}$$

and

$$\eta_{\text{Diesel}} = 1 - \frac{1}{(r)^{\gamma - 1}} \times \frac{1}{\gamma}\left\{\frac{\rho^{\gamma} - 1}{\rho - 1}\right\}$$

As the compression ratio is same,

$$\frac{V_1}{V_2} = \frac{V_1'}{V_2'} = r$$

If $\frac{V_4'}{V_3'} = r_1$, then cut-off ratio, $\rho = \frac{V_3'}{V_2'} = \frac{r}{r_1}$

Putting the value of ρ in η_{Diesel}, we get

$$\eta_{Diesel} = 1 - \frac{1}{(r)^{\gamma-1}} \times \frac{1}{\gamma}\left[\frac{\left(\frac{r}{r_1}\right)^{\gamma} - 1}{\frac{r}{r_1} - 1}\right]$$

From above equation, we observe

$$\frac{r}{r_1} > 1$$

Let $r_1 = r - \delta$, where δ is a small quantity.

Then $$\frac{r}{r_1} = \frac{r}{r-\delta} = \frac{r}{r\left(1-\frac{\delta}{r}\right)} = \left(1-\frac{\delta}{r}\right)^{-1} = 1 + \frac{\delta}{r} + \frac{\delta^2}{r^2} + \frac{\delta^3}{r^3} + \ldots\ldots$$

and $$\left(\frac{r}{r_1}\right)^{\gamma} = \frac{r^{\gamma}}{r^{\gamma}\left(1-\frac{\delta}{r}\right)^{\gamma}} = \left(1-\frac{\delta}{r}\right)^{-\gamma} = 1 + \frac{\gamma\delta}{r} + \frac{\gamma(\gamma+1)}{2!}\cdot\frac{\delta^2}{r^2} + \ldots\ldots$$

$\therefore$ $$\eta_{Diesel} = 1 - \frac{1}{(r)^{\gamma-1}} \times \frac{1}{\gamma}\left[\frac{\frac{\gamma\cdot\delta}{r} + \frac{\gamma(\gamma+1)}{2!}\cdot\frac{\delta^2}{r^2} + \ldots\ldots}{\frac{\delta}{r} + \frac{\delta^2}{r^2} + \ldots\ldots}\right]$$

$$= 1 - \frac{1}{(r)^{\gamma-1}}\left[\frac{\frac{\delta}{r} + \frac{\gamma+1}{2}\cdot\frac{\delta^2}{r^2} + \ldots\ldots}{\frac{\delta}{r} + \frac{\delta^2}{r^2} + \ldots\ldots}\right]$$

The ratio inside the bracket is greater than 1 since the co-efficients of terms δ^2/r^2 is greater than 1 in the numerator. Its means that something more is subtracted in case of diesel cycle than in Otto cycle.

Hence, *for same compression ratio* $\eta_{otto} > \eta_{diesel}$.

13.5. CONSTANT PRESSURE OR DIESEL CYCLE

This cycle was introduced by Dr. R. Diesel in 1897. It differs from Otto cycle in that *heat is supplied at constant pressure instead of at constant volume*. Fig. 13.15 (*a* and *b*) shows the *p-v* and *T-s* diagrams of this cycle respectively.

This cycle comprises of the following ***operations*** :

(*i*) 1-2.....*Adiabatic compression.*

(*ii*) 2-3.....*Addition of heat at constant pressure.*

(*iii*) 3-4.....*Adiabatic expansion.*

(*iv*) 4-1.....*Rejection of heat at constant volume.*

Point 1 represents that the cylinder is full of air. Let p_1, V_1 and T_1 be the corresponding pressure, volume and absolute temperature. The piston then compresses the air adiabatically (*i.e.*, pV^{γ} = constant) till the values become p_2, V_2 and T_2 respectively (at the end of the stroke) at point 2. Heat is then added from a hot body at a constant pressure. During this addition of heat let

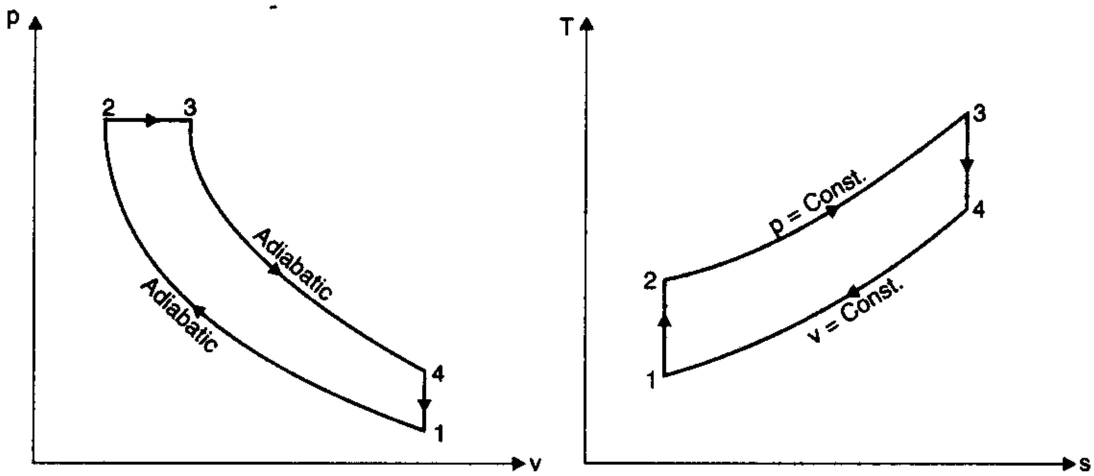


Fig. 13.15

volume increases from V_2 to V_3 and temperature T_2 to T_3, corresponding to point 3. This point (3) is called the ***point of cut-off***. The air then expands adiabatically to the conditions p_4, V_4 and T_4 respectively corresponding to point 4. Finally, the air rejects the heat to the cold body at constant volume till the point 1 where it returns to its original state.

Consider 1 kg of air.

Heat supplied at constant pressure $= c_p(T_3 - T_2)$

Heat rejected at constant volume $= c_v(T_4 - T_1)$

Work done = Heat supplied – heat rejected

$$= c_p(T_3 - T_2) - c_v(T_4 - T_1)$$

$$\therefore \quad \eta_{diesel} = \frac{\text{Work done}}{\text{Heat supplied}}$$

$$= \frac{c_p(T_3 - T_2) - c_v(T_4 - T_1)}{c_p(T_3 - T_2)}$$

$$= 1 - \frac{(T_4 - T_1)}{\gamma(T_3 - T_2)} \quad ...(i) \quad \left[\because \frac{c_p}{c_v} = \gamma\right]$$

Let compression ratio, $r = \frac{v_1}{v_2}$, and cut-off ratio, $\rho = \frac{v_3}{v_2}$ *i.e.*, $\frac{\text{Volume at cut-off}}{\text{Clearance volume}}$

Now, during *adiabatic compression 1-2,*

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = (r)^{\gamma - 1} \quad \text{or} \quad T_2 = T_1 \,.\, (r)^{\gamma - 1}$$

During *constant pressure process 2-3,*

$$\frac{T_3}{T_2} = \frac{v_3}{v_2} = \rho \quad \text{or} \quad T_3 = \rho \,.\, T_2 = \rho \,.\, T_1 \,.\, (r)^{\gamma - 1}$$

During *adiabatic expansion 3-4*

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma - 1}$$

$$= \left(\frac{r}{\rho}\right)^{\gamma-1} \qquad \left(\because \frac{v_4}{v_3} = \frac{v_1}{v_3} = \frac{v_1}{v_2} \times \frac{v_2}{v_3} = \frac{r}{\rho}\right)$$

$$\therefore \quad T_4 = \frac{T_3}{\left(\frac{r}{\rho}\right)^{\gamma-1}} = \frac{\rho \,.\, T_1 (r)^{\gamma-1}}{\left(\frac{r}{\rho}\right)^{\gamma-1}} = T_1 \,.\, \rho^{\gamma}$$

By inserting values of T_2, T_3 and T_4 in eqn. (*i*), we get

$$\eta_{diesel} = 1 - \frac{(T_1 \,.\, \rho^{\gamma} - T_1)}{\gamma\,(\rho \,.\, T_1 \,.\, (r)^{\gamma-1} - T_1 \,.\, (r)^{\gamma-1})} = 1 - \frac{(\rho^{\gamma} - 1)}{\gamma (r)^{\gamma-1} (\rho - 1)}$$

or

$$\eta_{diesel} = 1 - \frac{1}{\gamma (r)^{\gamma-1}} \left[\frac{\rho^{\gamma} - 1}{\rho - 1}\right] \qquad ...(13.7)$$

It may be observed that eqn. (13.7) for efficiency of diesel cycle is different from that of the Otto cycle only in bracketed factor. This factor is always greater than unity, because $\rho > 1$. Hence *for a given compression ratio, the Otto cycle is more efficient.*

The *net work* for diesel cycle can be expressed in terms of *pv* as follows :

$$W = p_2(v_3 - v_2) + \frac{p_3 v_3 - p_4 v_4}{\gamma - 1} - \frac{p_2 v_2 - p_1 v_1}{\gamma - 1}$$

$$= p_2 \,(\rho v_2 - v_2) + \frac{p_3 \rho v_2 - p_4 r v_2}{\gamma - 1} - \frac{p_2 v_2 - p_1 r v_2}{\gamma - 1}$$

$$\left[\begin{array}{l} \because \dfrac{v_3}{v_2} = \rho \quad \therefore \; v_3 = \rho v_2 \text{ and } \dfrac{v_1}{v_2} = r \quad \therefore \; v_1 = r v_2 \\ \text{But } v_4 = v_1 \quad \therefore \quad v_4 = r v_2 \end{array}\right]$$

$$= p_2 v_2 \,(\rho - 1) + \frac{p_3 \rho v_2 - p_4 r v_2}{\gamma - 1} - \frac{p_2 v_2 - p_1 r v_2}{\gamma - 1}$$

$$= \frac{v_2 \,[p_2 (\rho - 1)(\gamma - 1) + p_3 \rho - p_4 r - (p_2 - p_1 r)]}{\gamma - 1}$$

$$= \frac{v_2 \left[p_2 (\rho - 1)(\gamma - 1) + p_3 \left(\rho - \frac{p_4 r}{p_3}\right) - p_2 \left(1 - \frac{p_1 r}{p_2}\right)\right]}{\gamma - 1}$$

$$= \frac{p_2 v_2 [(\rho - 1)(\gamma - 1) + \rho - \rho^{\gamma} \,.\, r^{1-\gamma} - (1 - r^{1-\gamma})]}{\gamma - 1}$$

$$\left[\because \frac{p_4}{p_3} = \left(\frac{v_3}{v_4}\right)^{\gamma} = \left(\frac{\rho}{r}\right)^{\gamma} = \rho^{\gamma} r^{-\gamma}\right]$$

$$= \frac{p_1 v_1 r^{\gamma-1} [(\rho - 1)(\gamma - 1) + \rho - \rho^{\gamma} r^{1-\gamma} - (1 - r^{1-\gamma})}{\gamma - 1}$$

$$\left[\because \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} \text{ or } p_2 = p_1 \,.\, r^{\gamma} \text{ and } \frac{v_1}{v_2} = r \text{ or } v_2 = v_1 \, r^{-1}\right]$$

$$= \frac{p_1 v_1 r^{\gamma-1} [\gamma(\rho - 1) - r^{1-\gamma} (\rho^{\gamma} - 1)]}{(\gamma - 1)} \qquad ...(13.8)$$

Mean effective pressure p_m is given by :

$$p_m = \frac{p_1 v_1 r^{\gamma-1}[\gamma(\rho-1) - r^{1-\gamma}(\rho^\gamma - 1)]}{(\gamma-1)v_1\left(\frac{r-1}{r}\right)}$$

or
$$\mathbf{p_m} = \frac{p_1 r^\gamma [\gamma(\rho-1) - r^{1-\gamma}(\rho^\gamma - 1)]}{(\gamma-1)(r-1)} \quad ...(13.9)$$

Example 13.17. *A diesel engine has a compression ratio of 15 and heat addition at constant pressure takes place at 6% of stroke. Find the air standard efficiency of the engine.*

Take γ for air as 1.4.

Solution. Refer Fig. 13.16.

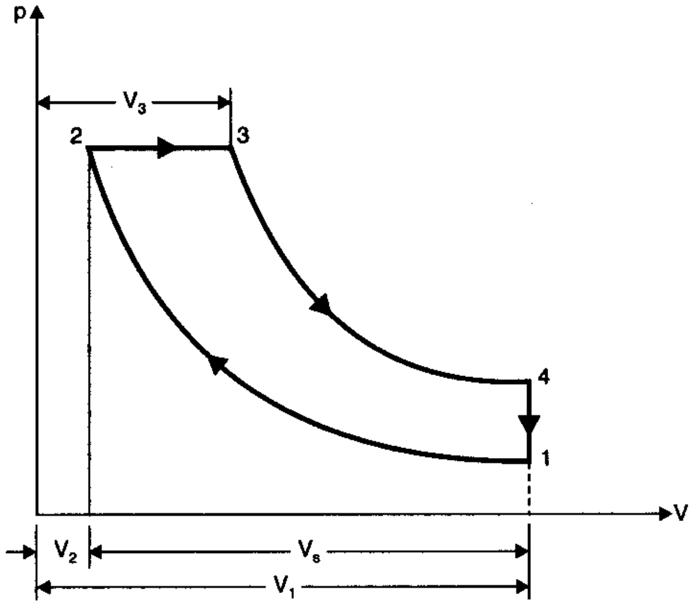


Fig. 13.16

Compression ratio, $r\left(=\frac{V_1}{V_2}\right) = 15$

γ for air = 1.4

Air standard efficiency of diesel cycle is given by

$$\eta_{diesel} = 1 - \frac{1}{\gamma(r)^{\gamma-1}}\left[\frac{\rho^\gamma - 1}{\rho - 1}\right] \quad ...(i)$$

where ρ = cut-off ratio = $\frac{V_3}{V_2}$

But $V_3 - V_2 = \frac{6}{100} V_s$ (V_s = stroke volume)

$= 0.06\,(V_1 - V_2) = 0.06\,(15\,V_2 - V_2)$

$= 0.84\,V_2$ or $V_3 = 1.84\,V_2$

$$\therefore \quad \rho = \frac{V_3}{V_2} = \frac{1.84\,V_2}{V_2} = 1.84$$

Putting the value in eqn. (*i*), we get

$$\eta_{diesel} = 1 - \frac{1}{1.4\,(15)^{1.4-1}}\left[\frac{(1.84)^{1.4} - 1}{1.84 - 1}\right]$$

$= 1 - 0.2417 \times 1.605 = $ **0.612 or 61.2%. (Ans.)**

Example 13.18. *The stroke and cylinder diameter of a compression ignition engine are 250 mm and 150 mm respectively. If the clearance volume is 0.0004 m³ and fuel injection takes place at constant pressure for 5 per cent of the stroke determine the efficiency of the engine. Assume the engine working on the diesel cycle.*

Solution. Refer Fig. 13.16.

Length of stroke, $L = 250$ mm $= 0.25$ m

Diameter of cylinder, $D = 150$ mm $= 0.15$ m

Clearance volume, $V_2 = 0.0004\ m^3$

Swept volume, $V_s = \pi/4\ D^2L = \pi/4 \times 0.15^2 \times 0.25 = 0.004418\ m^3$

Total cylinder volume

= Swept volume + clearance volume

$= 0.004418 + 0.0004 = 0.004818\ m^3$

Volume at point of cut-off, $V_3 = V_2 + \frac{5}{100} V_s$

$$= 0.0004 + \frac{5}{100} \times 0.004418 = 0.000621\ m^3$$

$\therefore$ Cut-off ratio, $\rho = \frac{V_3}{V_2} = \frac{0.000621}{0.0004} = 1.55$

Compression ratio, $r = \frac{V_1}{V_2} = \frac{V_s + V_2}{V_2} = \frac{0.004418 + 0.0004}{0.0004} = 12.04$

Hence,

$$\eta_{diesel} = 1 - \frac{1}{\gamma (r)^{\gamma-1}}\left[\frac{\rho^\gamma - 1}{\rho - 1}\right] = 1 - \frac{1}{1.4 \times (12.04)^{1.4-1}}\left[\frac{(1.55)^{1.4} - 1}{1.55 - 1}\right]$$

$= 1 - 0.264 \times 1.54 = $ **0.593 or 59.3%. (Ans.)**

Example 13.19. *Calculate the percentage loss in the ideal efficiency of a diesel engine with compression ratio 14 if the fuel cut-off is delayed from 5% to 8%.*

Solution. Let the clearance volume (V_2) be unity.

Then, compression ratio, $r = 14$

Now, when the fuel is cut off at 5%, we have

$$\frac{\rho - 1}{r - 1} = \frac{5}{100} \quad \text{or} \quad \frac{\rho - 1}{14 - 1} = 0.05 \quad \text{or} \quad \rho - 1 = 13 \times 0.05 = 0.65$$

$\therefore \quad \rho = 1.65$

$$\eta_{diesel} = 1 - \frac{1}{\gamma (r)^{\gamma-1}}\left[\frac{\rho^\gamma - 1}{\rho - 1}\right] = 1 - \frac{1}{1.4 \times (14)^{1.4-1}}\left[\frac{(1.65)^{1.4} - 1}{1.65 - 1}\right]$$

$$= 1 - 0.248 \times 1.563 = 0.612 \quad \text{or} \quad 61.2\%$$

When the fuel is cut-off at 8%, we have

$$\frac{\rho - 1}{r - 1} = \frac{8}{100} \quad \text{or} \quad \frac{\rho - 1}{14 - 1} = \frac{8}{100} = 0.08$$

$$\therefore \quad \rho = 1 + 1.04 = 2.04$$

$$\eta_{\text{diesel}} = 1 - \frac{1}{\gamma (r)^{\gamma - 1}}\left[\frac{\rho^{\gamma} - 1}{\rho - 1}\right] = 1 - \frac{1}{1.4 \times (14)^{1.4 - 1}}\left[\frac{(2.04)^{1.4} - 1}{2.04 - 1}\right]$$

$$= 1 - 0.248 \times 1.647 = 0.591 \quad \text{or} \quad 59.1\%.$$

Hence percentage loss in efficiency due to delay in fuel cut off

$$= 61.2 - 59.1 = \mathbf{2.1\%.} \quad \textbf{(Ans.)}$$

Example 13.20. *The mean effective pressure of a Diesel cycle is 7.5 bar and compression ratio is 12.5. Find the percentage cut-off of the cycle if its initial pressure is 1 bar.*

Solution. Mean effective pressure, $p_m = 7.5$ bar

Compression ratio, $r = 12.5$

Initial pressure, $p_1 = 1$ bar

Refer Fig. 13.15.

The mean effective pressure is given by

$$p_m = \frac{p_1 r^{\gamma}[\gamma(\rho - 1) - r^{1-\gamma}(\rho^{\gamma} - 1)]}{(\gamma - 1)(r - 1)}$$

$$7.5 = \frac{1 \times (12.5)^{1.4}[1.4(\rho - 1) - (12.5)^{1-1.4}(\rho^{1.4} - 1)]}{(1.4 - 1)(12.5 - 1)}$$

$$7.5 = \frac{34.33[1.4\,\rho - 1.4 - 0.364\rho^{1.4} + 0.364]}{4.6}$$

$$7.5 = 7.46\,(1.4\,\rho - 1.036 - 0.364\,\rho^{1.4})$$

$$1.005 = 1.4\,\rho - 1.036 - 0.364\,\rho^{1.4}$$

or $$2.04 = 1.4\,\rho - 0.364\,\rho^{1.4} \quad \text{or} \quad 0.346\,\rho^{1.4} - 1.4\,\rho + 2.04 = 0$$

Solving by trial and error method, we get

$$\rho = 2.24$$

$$\therefore \quad \%\ \textit{cut-off} = \frac{\rho - 1}{r - 1} \times 100 = \frac{2.24 - 1}{12.5 - 1} \times 100 = \mathbf{10.78\%.} \quad \textbf{(Ans.)}$$

☞**Example 13.21.** *An engine with 200 mm cylinder diameter and 300 mm stroke works on theoretical Diesel cycle. The initial pressure and temperature of air used are 1 bar and 27°C. The cut-off is 8% of the stroke. Determine :*

(i) Pressures and temperatures at all salient points.

(ii) Theoretical air standard efficiency.

(iii) Mean effective pressure.

(iv) Power of the engine if the working cycles per minute are 380.

Assume that compression ratio is 15 and working fluid is air.

Consider all conditions to be ideal.

Solution. Refer Fig. 13.17.

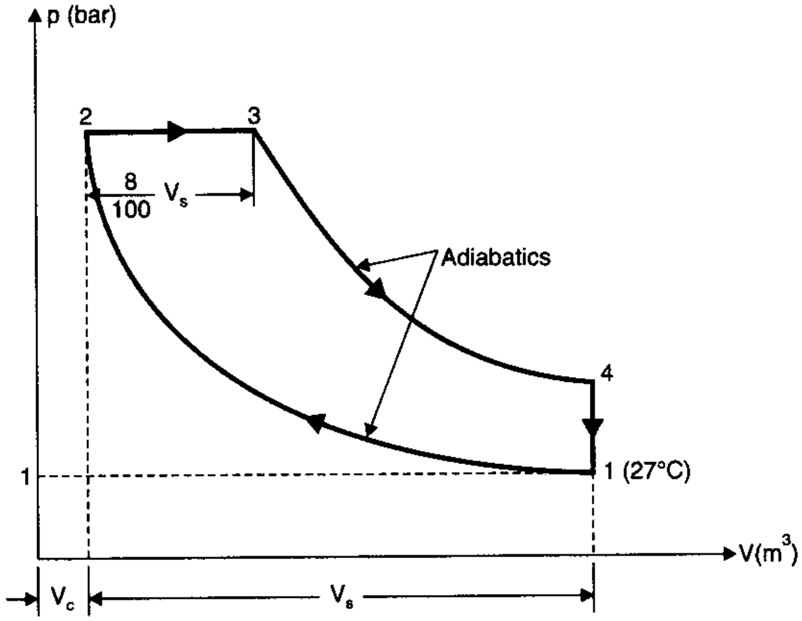


Fig. 13.17

Cylinder diameter,	D = 200 mm or 0.2 m
Stroke length,	L = 300 mm or 0.3 m
Initial pressure,	p_1 = 1.0 bar
Initial temperature,	T_1 = 27 + 273 = 300 K
Cut-off	$= \frac{8}{100} V_s = 0.08\ V_s$

(*i*) **Pressures and temperatures at salient points :**

Now, stroke volume, $V_s = \pi/4\ D^2L = \pi/4 \times 0.2^2 \times 0.3 = 0.00942\ m^3$

$$V_1 = V_s + V_c = V_s + \frac{V_s}{r-1} \qquad \left[\because\ V_c = \frac{V_s}{r-1}\right]$$

$$= V_s\left(1 + \frac{1}{r-1}\right) = \frac{r}{r-1} \times V_s$$

i.e.,
$$V_1 = \frac{15}{15-1} \times V_s = \frac{15}{14} \times 0.00942 = \mathbf{0.0101\ m^3.} \quad \textbf{(Ans.)}$$

Mass of the air in the cylinder can be calculated by using the gas equation,

$$p_1V_1 = mRT_1$$

$$m = \frac{p_1V_1}{RT_1} = \frac{1 \times 10^5 \times 0.0101}{287 \times 300} = 0.0117 \text{ kg/cycle}$$

For the *adiabatic (or isentropic) process 1-2*

$$p_1V_1^{\gamma} = p_2V_2^{\gamma} \quad \text{or} \quad \frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = (r)^{\gamma}$$

$\therefore$ $\quad \mathbf{p_2} = p_1 \cdot (r)^{\gamma} = 1 \times (15)^{1.4} = \mathbf{44.31\ bar.}$ **(Ans.)**

Also, $\quad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r)^{\gamma-1} = (15)^{1.4-1} = 2.954$

$\therefore$ $\quad \mathbf{T_2} = T_1 \times 2.954 = 300 \times 2.954 = \mathbf{886.2\ K.}$ **(Ans.)**

$$\mathbf{V_2} = V_c = \frac{V_s}{r-1} = \frac{0.00942}{15-1} = \mathbf{0.0006728\ m^3.} \text{ (Ans.)}$$

$$\mathbf{p_2} = p_3 = \mathbf{44.31\ bar.} \text{ (Ans.)}$$

% cut-off ratio $\quad = \frac{\rho - 1}{r - 1}$

$$\frac{8}{100} = \frac{\rho - 1}{15 - 1}$$

i.e., $\quad \rho = 0.08 \times 14 + 1 = 2.12$

$\therefore$ $\quad V_3 = \rho\, V_2 = 2.12 \times 0.0006728 = \mathbf{0.001426\ m^3.}$ **(Ans.)**

$$\left[\begin{array}{l} V_3 \text{ can also be calculated as follows :} \\ V_3 = 0.08V_s + V_c = 0.08 \times 0.00942 + 0.0006728 = 0.001426 \text{ m}^3 \end{array}\right]$$

For the *constant pressure process 2-3,*

$$\frac{V_3}{T_3} = \frac{V_2}{T_2}$$

$\therefore$ $\quad \mathbf{T_3} = T_2 \times \frac{V_3}{V_2} = 886.2 \times \frac{0.001426}{0.0006728} = \mathbf{1878.3\ K.}$ **(Ans.)**

For the *isentropic process 3-4,*

$$p_3V_3^{\gamma} = p_4V_4^{\gamma}$$

$$p_4 = p_3 \times \left(\frac{V_3}{V_4}\right)^{\gamma} = p_3 \times \frac{1}{(7.07)^{1.4}} \qquad \left[\begin{array}{l} \because \ \frac{V_4}{V_3} = \frac{V_4}{V_2} \times \frac{V_2}{V_3} = \frac{V_1}{V_2} \times \frac{V_2}{V_3} \\ \quad = \frac{r}{\rho}, \ \because \ V_4 = V_1 = \frac{15}{2.12} = 7.07 \end{array}\right]$$

$$= \frac{44.31}{(7.07)^{1.4}} = \mathbf{2.866\ bar.} \text{ (Ans.)}$$

Also, $\quad \frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \left(\frac{1}{7.07}\right)^{1.4-1} = 0.457$

$\therefore$ $\quad \mathbf{T_4} = T_3 \times 0.457 = 1878.3 \times 0.457 = \mathbf{858.38\ K.}$ **(Ans.)**

$$\mathbf{V_4} = V_1 = \mathbf{0.0101\ m^3.} \text{ (Ans.)}$$

(*ii*) **Theoretical air standard efficiency :**

$$\eta_{diesel} = 1 - \frac{1}{\gamma(r)^{\gamma-1}}\left[\frac{\rho^{\gamma} - 1}{\rho - 1}\right] = 1 - \frac{1}{1.4\,(15)^{1.4-1}}\left[\frac{(2.12)^{1.4} - 1}{2.12 - 1}\right]$$

$$= 1 - 0.2418 \times 1.663 = \mathbf{0.598\ or\ 59.8\%.} \text{ (Ans.)}$$

(*iii*) **Mean effective pressure, p_m :**

Mean effective pressure of Diesel cycle is given by

$$p_m = \frac{p_1(r)^\gamma[\gamma(\rho-1) - r^{1-\gamma}(\rho^\gamma - 1)}{(\gamma-1)(r-1)}$$

$$= \frac{1 \times (15)^{1.4}[1.4(2.12-1) - (15)^{1-1.4}(2.12^{1.4}-1)]}{(1.4-1)(15-1)}$$

$$= \frac{44.31[1.568 - 0.338 \times 1.863]}{0.4 \times 14} = \textbf{7.424 bar.} \quad \textbf{(Ans.)}$$

(*iv*) **Power of the engine, P :**

Work done per cycle $= p_m V_s = \frac{7.424 \times 10^5 \times 0.00942}{10^3} = 6.99$ kJ/cycle

Work done per second = Work done per cycle × no. of cycles per second

= 6.99 × 380/60 = 44.27 kJ/s = 44.27 kW

Hence *power of the engine* = **44.27 kW. (Ans.)**

Example 13.22. *The volume ratios of compression and expansion for a diesel engine as measured from an indicator diagram are 15.3 and 7.5 respectively. The pressure and temperature at the beginning of the compression are 1 bar and 27°C.*

Assuming an ideal engine, determine the mean effective pressure, the ratio of maximum pressure to mean effective pressure and cycle efficiency.

Also find the fuel consumption per kWh if the indicated thermal efficiency is 0.5 of ideal efficiency, mechanical efficiency is 0.8 and the calorific value of oil 42000 kJ/kg.

Assume for air : c_p = 1.005 kJ/kg K ; c_v = 0.718 kJ/kg K, γ = 1.4. **(U.P.S.C., 1996)**

Solution. Refer Fig. 13.18. *Given :* $\frac{V_1}{V_2} = 15.3$; $\frac{V_4}{V_3} = 7.5$

p_1 = 1 bar ; T_1 = 27 + 273 = 300 K ; $\eta_{th(I)} = 0.5 \times \eta_{air\text{-}standard}$; $\eta_{mech.} = 0.8$; C = 42000 kJ/kg.

The cycle is shwon in Fig. 13.18, the subscripts denote the respective points in the cycle.

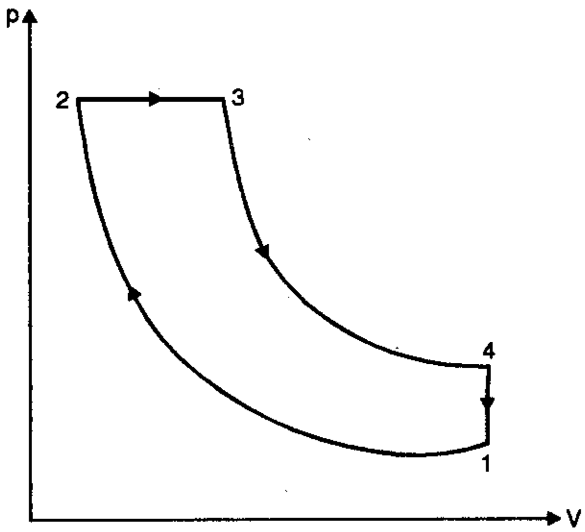


Fig. 13.18. Diesel cycle.

Mean effective pressure, p_m :

$$p_m = \frac{\text{Work done by the cycle}}{\text{Swept volume}}$$

Work done = Heat added – heat rejected

Heat added $= mc_p\,(T_3 - T_2)$, and

Heat rejected $= mc_v\,(T_4 - T_1)$

Now assume air as a perfect gas and mass of oil in the air-fuel mixture is negligible and is not taken into account.

Process 1-2 is an *adiabatic compression process*, thus

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} \quad \text{or} \quad T_2 = T_1 \times \left(\frac{V_1}{V_2}\right)^{1.4-1} \quad \text{(since } \gamma = 1.4)$$

or $$T_2 = 300 \times (15.3)^{0.4} = 893.3 \text{ K}$$

Also, $$p_1V_1^{\gamma} = p_2V_2^{\gamma} \Rightarrow p_2 = p_1 \times \left(\frac{V_1}{V_2}\right)^{\gamma} = 1 \times (15.3)^{1.4} = 45.56 \text{ bar}$$

Process 2-3 is a *constant pressure process*, hence

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \Rightarrow T_3 = \frac{V_3T_2}{V_2} = 2.04 \times 893.3 = 1822.3 \text{ K}$$

Assume that the volume at point 2 (V_2) is 1 m^3. Thus the mass of air involved in the process,

$$m = \frac{p_2V_2}{RT_2} = \frac{45.56 \times 10^5 \times 1}{287 \times 893.3} = 17.77 \text{ kg}$$

$$\left[\because \frac{V_4}{V_3} = \frac{V_1}{V_3} = \frac{V_1}{V_2} \times \frac{V_2}{V_3} \quad \text{or} \quad \frac{V_3}{V_2} = \frac{V_1}{V_2} \times \frac{V_3}{V_4} = \frac{15.3}{7.5} = 2.04\right]$$

Process 3-4 is an *adiabatic expansion process*, thus

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma - 1} = \left(\frac{1}{7.5}\right)^{1.4-1} = 0.4466$$

or $$T_4 = 1822.3 \times 0.4466 = 813.8 \text{ K}$$

$\therefore$ Work done $= mc_p\,(T_3 - T_2) - mc_v\,(T_4 - T_1)$

$= 17.77\,[1.005\,(1822.3 - 893.3) - 0.718\,(813.8 - 300)] = 10035$ kJ

$\therefore$ $$\mathbf{p_m} = \frac{\text{Work done}}{\text{Swept volume}} = \frac{10035}{(V_1 - V_2)} = \frac{10035}{(15.3V_2 - V_2)} = \frac{10035}{14.3}$$

$= 701.7$ kN/m^2 $=$ **7.017 bar. (Ans.)**

($\because V_2 = 1$ m^3 assumed)

Ratio of maximum pressure to mean effective pressure

$$= \frac{p_2}{p_m} = \frac{45.56}{7.017} = \mathbf{6.49.} \quad \textbf{(Ans.)}$$

Cycle efficiency, η_{cycle} :

$$\eta_{cycle} = \frac{\text{Work done}}{\text{Heat supplied}}$$

$$= \frac{10035}{mc_p\,(T_3 - T_2)} = \frac{10035}{17.77 \times 1.005\,(1822.3 - 897.3)} = \mathbf{0.6048 \ \text{or} \ 60.48\%.} \quad \textbf{(Ans.)}$$

Fuel consumption per kWh ; m_f :

$\eta_{th(I)} = 0.5\ \eta_{cycle} = 0.5 \times 0.6048 = 0.3024$ or 30.24%

$$\eta_{th(B)} = 0.3024 \times 0.8 = 0.242$$

Also,

$$\eta_{th(B)} = \frac{\text{B.P.}}{m_f \times C} = \frac{1}{\frac{m_f}{3600} \times 42000} = \frac{3600}{m_f \times 42000}$$

or

$$0.242 = \frac{3600}{m_f \times 42000}$$

or

$$m_f = \frac{3600}{0.242 \times 42000} = \mathbf{0.354\ kg/kWh.} \quad \textbf{(Ans.)}$$

13.6. DUAL COMBUSTION CYCLE

This cycle (also called the *limited pressure cycle or mixed cycle*) is a combination of Otto and Diesel cycles, in a way, that heat is added partly at constant volume and partly at constant pressure ; *the advantage of which is that more time is available to fuel (which is injected into the engine cylinder before the end of compression stroke) for combustion. Because of lagging characteristics of fuel this cycle is invariably used for diesel and hot spot ignition engines.*

The dual combustion cycle (Fig. 13.19) consists of the following **operations** :

(*i*) 1-2—*Adiabatic compression*

(*ii*) 2-3—*Addition of heat at constant volume*

(*iii*) 3-4—*Addition of heat at constant pressure*

(*iv*) 4-5—*Adiabatic expansion*

(*v*) 5-1—*Rejection of heat at constant volume.*

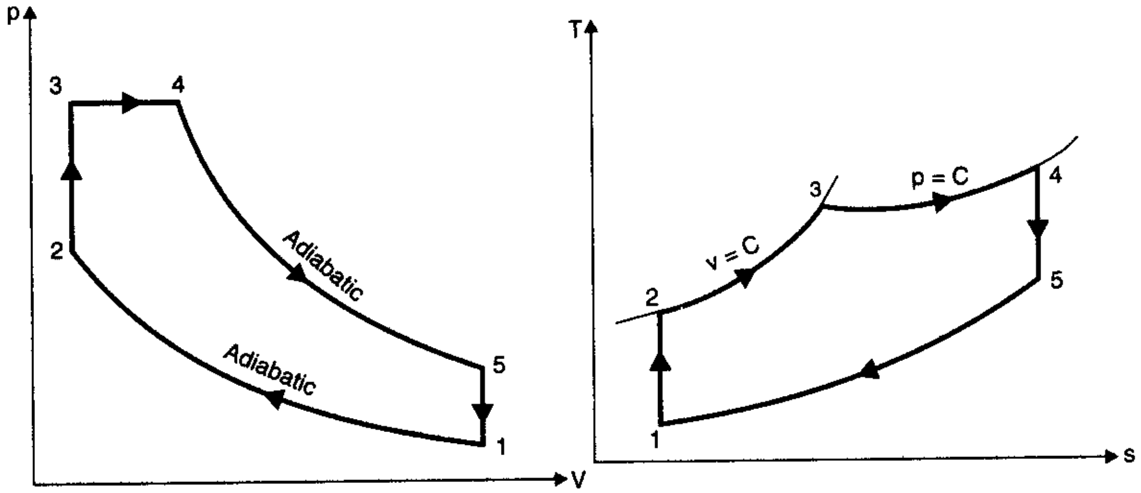


Fig. 13.19.

Consider 1 kg of air.

Total heat supplied = Heat supplied during the operation 2-3 + heat supplied during the operation 3-4

$$= c_v(T_3 - T_2) + c_p(T_4 - T_3)$$

Heat rejected during operation 5-1 $= c_v(T_5 - T_1)$

Work done = Heat supplied – heat rejected

$$= c_v(T_3 - T_2) + c_p(T_4 - T_3) - c_v(T_5 - T_1)$$

$$\eta_{\text{dual}} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{c_v(T_3 - T_2) + c_p(T_4 - T_3) - c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)}$$

$$= 1 - \frac{c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)}$$

$$= 1 - \frac{c_v(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)} \quad ...(i) \quad \left(\because \gamma = \frac{c_p}{c_v}\right)$$

Compression ratio, $r = \dfrac{v_1}{v_2}$

During *adiabatic compression process 1-2,*

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = (r)^{\gamma - 1} \quad ...(ii)$$

During *constant volume heating process,*

$$\frac{p_3}{T_3} = \frac{p_2}{T_2}$$

or $\dfrac{T_3}{T_2} = \dfrac{p_3}{p_2} = \beta$, where β is known as **pressure or explosion ratio.**

or $$T_2 = \frac{T_3}{\beta} \quad ...(iii)$$

During *adiabatic expansion process,*

$$\frac{T_4}{T_5} = \left(\frac{v_5}{v_4}\right)^{\gamma - 1}$$

$$= \left(\frac{r}{\rho}\right)^{\gamma - 1} \quad ...(iv)$$

$$\left(\because \frac{v_5}{v_4} = \frac{v_1}{v_4} = \frac{v_1}{v_2} \times \frac{v_2}{v_4} = \frac{v_1}{v_2} \times \frac{v_3}{v_4} = \frac{r}{\rho}, \rho \text{ being the cut-off ratio}\right)$$

During *constant pressure heating process,*

$$\frac{v_3}{T_3} = \frac{v_4}{T_4}$$

$$T_4 = T_3 \frac{v_4}{v_3} = \rho\, T_3 \quad ...(v)$$

Putting the value of T_4 in the eqn. (iv), we get

$$\frac{\rho T_3}{T_5} = \left(\frac{r}{\rho}\right)^{\gamma - 1} \quad \text{or} \quad T_5 = \rho \,.\, T_3 \,.\, \left(\frac{\rho}{r}\right)^{\gamma - 1}$$

Putting the value of T_2 in eqn. (*ii*), we get

$$\frac{\frac{T_3}{\beta}}{T_1} = (r)^{\gamma - 1}$$

$$T_1 = \frac{T_3}{\beta} \cdot \frac{1}{(r)^{\gamma - 1}}$$

Now inserting the values of T_1, T_2, T_4 and T_5 in eqn. (*i*), we get

$$\eta_{dual} = 1 - \frac{\left[\rho . T_3\left(\frac{\rho}{r}\right)^{\gamma - 1} - \frac{T_3}{\beta} \cdot \frac{1}{(r)^{\gamma - 1}}\right]}{\left[\left(T_3 - \frac{T_3}{\beta}\right) + \gamma(\rho T_3 - T_3)\right]} = 1 - \frac{\frac{1}{(r)^{\gamma - 1}}\left(\rho^{\gamma} - \frac{1}{\beta}\right)}{\left(1 - \frac{1}{\beta}\right) + \gamma(\rho - 1)}$$

i.e., $$\eta_{dual} = 1 - \frac{1}{(r)^{\gamma - 1}} \cdot \frac{(\beta . \rho^{\gamma} - 1)}{[(\beta - 1) + \beta\gamma(\rho - 1)]} \quad ...(13.10)$$

Work done is given by,

$$W = p_3(v_4 - v_3) + \frac{p_4v_4 - p_5v_5}{\gamma - 1} - \frac{p_2v_2 - p_1v_1}{\gamma - 1}$$

$$= p_3v_3(\rho - 1) + \frac{(p_4\rho v_3 - p_5 r v_3) - (p_2v_3 - p_1 r v_3)}{\gamma - 1}$$

$$= \frac{p_3v_3(\rho - 1)(\gamma - 1) + p_4v_3\left(\rho - \frac{p_5}{p_4}r\right) - p_2v_3\left(1 - \frac{p_1}{p_2}r\right)}{\gamma - 1}$$

Also $$\frac{p_5}{p_4} = \left(\frac{v_4}{v_5}\right)^{\gamma} = \left(\frac{\rho}{r}\right)^{\gamma} \quad \text{and} \quad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = r^{\gamma}$$

also, $p_3 = p_4$, $v_2 = v_3$, $v_5 = v_1$

$\therefore$ $$W = \frac{v_3[p_3(\rho - 1)(\gamma - 1) + p_3(\rho - \rho^{\gamma}r^{1-\gamma}) - p_2(1 - r^{1-\gamma})]}{(\gamma - 1)}$$

$$= \frac{p_2v_2[\beta(\rho - 1)(\gamma - 1) + \beta(\rho - \rho^{\gamma}r^{1-\gamma}) - (1 - r^{1-\gamma})]}{(\gamma - 1)}$$

$$= \frac{p_1(r)^{\gamma}v_1/r[\beta\gamma(\rho - 1) + (\beta - 1) - r^{1-\gamma}(\beta\rho^{\gamma} - 1)]}{\gamma - 1}$$

$$= \frac{p_1v_1r^{\gamma - 1}[\beta\gamma(\rho - 1) + (\beta - 1) - r^{\gamma - 1}(\beta\rho^{\gamma} - 1)]}{\gamma - 1} \quad ...(13.11)$$

Mean effective pressure (p_m) is given by,

$$p_m = \frac{W}{v_1 - v_2} = \frac{W}{v_1\left(\frac{r - 1}{r}\right)} = \frac{p_1v_1[r^{1-\gamma}\beta\gamma(\rho - 1) + (\beta - 1) - r^{1-\gamma}(\beta\rho^{\gamma} - 1)]}{(\gamma - 1)v_1\left(\frac{r - 1}{r}\right)}$$

$$\mathbf{p_m} = \frac{p_1(r)^{\gamma}[\beta(\rho - 1) + (\beta - 1) - r^{1-\gamma}(\beta\rho^{\gamma} - 1)]}{(\gamma - 1)(r - 1)} \quad ...(13.12)$$

Example 13.23. *The swept volume of a diesel engine working on dual cycle is 0.0053 m^3 and clearance volume is 0.00035 m^3. The maximum pressure is 65 bar. Fuel injection ends at 5 per cent of the stroke. The temperature and pressure at the start of the compression are 80°C and 0.9 bar. Determine the air standard efficiency of the cycle. Take γ for air = 1.4.*

Solution. Refer Fig. 13.20.

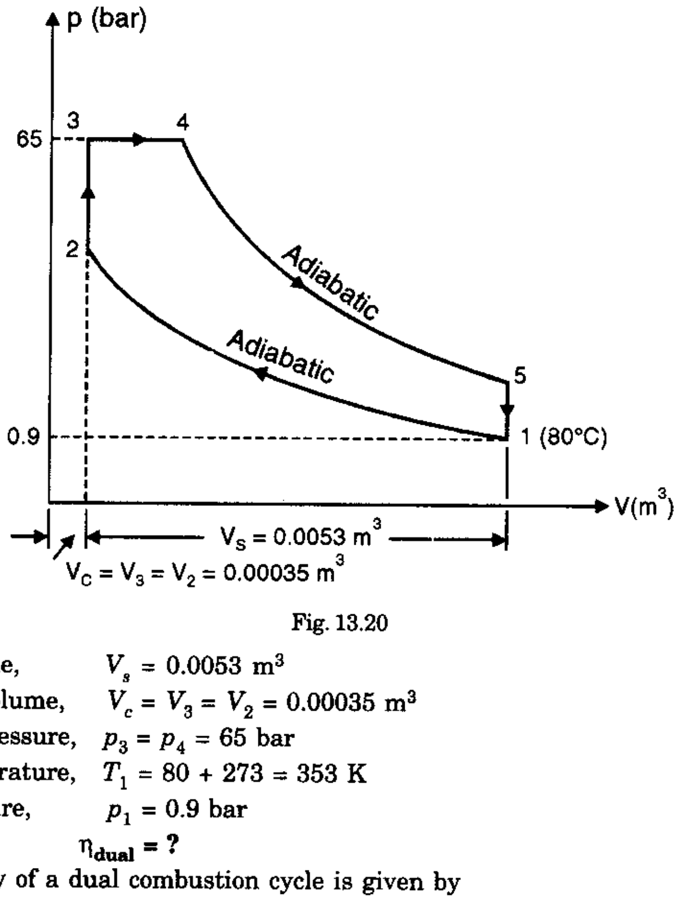


Fig. 13.20

Swept volume, V_s = 0.0053 m^3

Clearance volume, $V_c = V_3 = V_2$ = 0.00035 m^3

Maximum pressure, $p_3 = p_4$ = 65 bar

Initial temperature, T_1 = 80 + 273 = 353 K

Initial pressure, p_1 = 0.9 bar

η_{dual} = ?

The efficiency of a dual combustion cycle is given by

$$\eta_{dual} = 1 - \frac{1}{(r)^{\gamma-1}}\left[\frac{\beta \cdot \rho^{\gamma} - 1}{(\beta - 1) + \beta\gamma(\rho - 1)}\right] \quad ...(i)$$

Compression ratio, $$r = \frac{V_1}{V_2} = \frac{V_s + V_c}{V_c} = \frac{0.0053 + 0.00035}{0.00035} = 16.14$$

[$\because$ $V_2 = V_c$ = Clearance volume]

Cut-off ratio, $$\rho = \frac{V_4}{V_3} = \frac{\frac{5}{100} V_s + V_3}{V_3} = \frac{0.05V_s + V_c}{V_c} \quad (\because\ V_2 = V_3 = V_c)$$

$$= \frac{0.05 \times 0.0053 + 0.00035}{0.00035} = 1.757 \text{ say } 1.76$$

Also during the *compression operation 1-2,*

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

or $$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = (16.14)^{1.4} = 49.14$$

or $$p_2 = p_1 \times 49.14 = 0.9 \times 49.14 = 44.22 \text{ bar}$$

Pressure or explosion ratio, $\beta = \frac{p_3}{p_2} = \frac{65}{44.22} = 1.47$

Putting the value of r, ρ and β in eqn. (*i*), we get

$$\eta_{dual} = 1 - \frac{1}{(16.14)^{1.4-1}}\left[\frac{1.47 \times (1.76)^{1.4} - 1}{(1.47 - 1) + 1.47 \times 1.4\,(1.76 - 1)}\right]$$

$$= 7 - 0.328\left[\frac{3.243 - 1}{0.47 + 1.564}\right] = \mathbf{0.6383 \text{ or } 63.83\%.} \quad \textbf{(Ans.)}$$

Example 13.24. *An oil engine working on the dual combustion cycle has a compression ratio 14 and the explosion ratio obtained from an indicator card is 1.4. If the cut-off occurs at 6 per cent of stroke, find the ideal efficiency. Take γ for air = 1.4.*

Solution. Refer Fig. 13.19.

Compression ratio, $r = 14$

Explosion ratio, $\beta = 1.4$

If ρ is the cut-off ratio, then $\frac{\rho - 1}{r - 1} = \frac{6}{100}$ or $\frac{\rho - 1}{14 - 1} = 0.06$

$\therefore$ $\rho = 1.78$

Ideal efficiency is given by

$$\eta_{ideal\ or\ dual} = 1 - \frac{1}{(r)^{\gamma-1}}\left[\frac{(\beta\rho^{\gamma} - 1)}{(\beta - 1) + \beta\gamma(\rho - 1)}\right]$$

$$= 1 - \frac{1}{(14)^{1.4-1}}\left[\frac{1.4 \times (1.78)^{1.4} - 1}{(1.4 - 1) + 1.4 \times 1.4(1.78 - 1)}\right]$$

$$= 1 - 0.348\left[\frac{3.138 - 1}{0.4 + 1.528}\right] = \mathbf{0.614 \text{ or } 61.4\%.} \quad \textbf{(Ans.)}$$

Example 13.25. *The compression ratio for a single-cylinder engine operating on dual cycle is 9. The maximum pressure in the cylinder is limited to 60 bar. The pressure and temperature of the air at the beginning of the cycle are 1 bar and 30°C. Heat is added during constant pressure process upto 4 per cent of the stroke. Assuming the cylinder diameter and stroke length as 250 mm and 300 mm respectively, determine :*

(*i*) *The air standard efficiency of the cycle.*

(*ii*) *The power developed if the number of working cycles are 3 per second.*

Take for air c_v = 0.71 kJ/kg K and : c_p = 1.0. kJ/kg K

Solution. Refer Fig. 13.21.

Cylinder diameter, D = 250 mm = 0.25 m

Compression ratio, $r = 9$

Stroke length, L = 300 mm = 0.3 m

Initial pressure, $p_1 = 1$ bar
Initial temperature, $T_1 = 30 + 273 = 303$ K
Maximum pressure, $p_3 = p_4 = 60$ bar
Cut-off = 4% of stroke volume
Number of working cycles/sec. = 3.

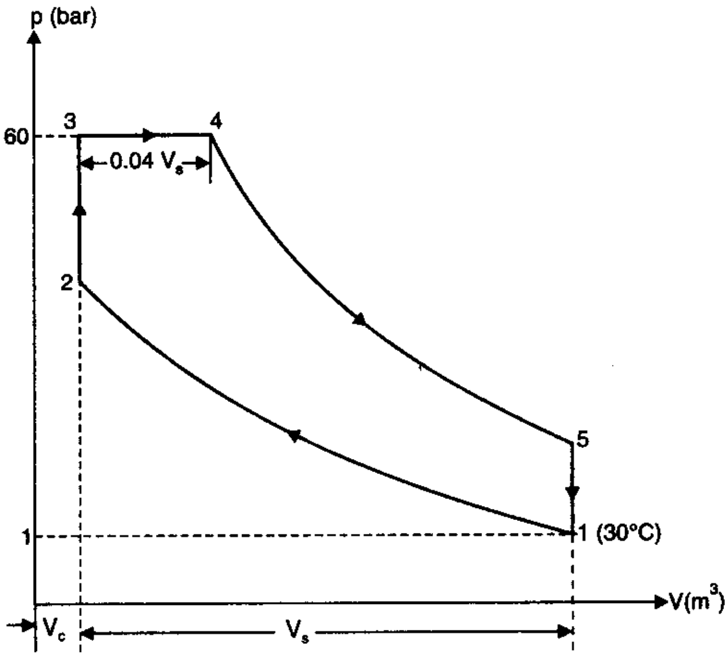


Fig. 13.21

(*i*) **Air standard efficiency :**

Now, swept volume, $V_s = \pi/4\ D^2L = \pi/4 \times 0.25^2 \times 0.3$
$= 0.0147\ m^3$

Also, compression ratio, $r = \dfrac{V_s + V_c}{V_c}$

i.e., $$9 = \frac{0.0147 + V_c}{V_c}$$

$\therefore$ $$V_c = \frac{0.0147}{8} = 0.0018\ m^3$$

$\therefore$ $$V_1 = V_s + V_c = 0.0147 + 0.0018 = 0.0165\ m^3$$

For the *adiabatic (or isentropic) process 1-2,*

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

$$p_2 = p_1 \times \left(\frac{V_1}{V_2}\right)^{\gamma} = 1 \times (r)^{\gamma} = 1 \times (9)^{1.4} = 21.67 \text{ bar}$$

Also, $$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = (r)^{\gamma - 1} = (9)^{1.4 - 1} = (9)^{0.4} = 2.408$$

$\therefore \quad T_2 = T_1 \times 2.408 = 303 \times 2.408 = 729.6$ K

For the *constant volume process 2-3,*

$$\frac{T_3}{p_3} = \frac{T_2}{p_2}$$

$$\therefore \quad T_3 = T_2 \cdot \frac{p_3}{p_2} = 729.6 \times \frac{60}{21.67} = 2020 \text{ K}$$

Also, $\quad \dfrac{\rho - 1}{r - 1} = \dfrac{4}{100}$ or 0.04

$$\therefore \quad \frac{\rho - 1}{9 - 1} = 0.04 \text{ or } \rho = 1.32$$

For the *constant pressure process 3-4,*

$$\frac{V_4}{T_4} = \frac{V_3}{T_3} \text{ or } \frac{T_4}{T_3} = \frac{V_4}{V_3} = \rho$$

$$\therefore \quad T_4 = T_3 \times \rho = 2020 \times 1.32 = 2666.4 \text{ K}$$

Also expansion ratio, $\quad \dfrac{V_5}{V_4} = \dfrac{V_5}{V_2} \times \dfrac{V_2}{V_4} = \dfrac{V_1}{V_2} \times \dfrac{V_3}{V_4} = \dfrac{r}{\rho} \quad [\because V_5 = V_1 \text{ and } V_2 = V_3]$

For *adiabatic process 4-5,*

$$\frac{T_5}{T_4} = \left(\frac{V_4}{V_5}\right)^{\gamma - 1} = \left(\frac{\rho}{r}\right)^{\gamma - 1}$$

$$\therefore \quad T_5 = T_4 \times \left(\frac{\rho}{r}\right)^{\gamma - 1} = 2666.4 \times \left(\frac{1.32}{9}\right)^{1.4 - 1} = 1237 \text{ K}$$

Also $\quad p_4 V_4^{\gamma} = p_5 V_5^{\gamma}$

$$p_5 = p_4 \cdot \left(\frac{V_4}{V_5}\right)^{\gamma} = 60 \times \left(\frac{r}{\rho}\right)^{\gamma} = 60 \times \left(\frac{1.32}{9}\right)^{1.4} = 4.08 \text{ bar}$$

Heat supplied, $\quad Q_s = c_v(T_3 - T_2) + c_p(T_4 - T_3)$

$= 0.71\ (2020 - 729.6) + 1.0\ (2666.4 - 2020) = 1562.58$ kJ/kg

Heat rejected, $\quad Q_r = c_v\ (T_5 - T_1)$

$= 0.71\ (1237 - 303) = 663.14$ kJ/kg

$$\eta_{\text{air-standard}} = \frac{Q_s - Q_r}{Q_s} = \frac{1562.85 - 663.14}{1562.58} = \mathbf{0.5756 \text{ or } 57.56\%}. \quad \textbf{(Ans.)}$$

(*ii*) **Power developed by the engine, P :**

Mass of air in the cycle is given by

$$m = \frac{p_1 V_1}{RT_1} = \frac{1 \times 10^5 \times 0.0165}{287 \times 303} = 0.0189 \text{ kg}$$

$\therefore$ Work done per cycle $= m(Q_s - Q_r)$

$= 0.0189\ (1562.58 - 663.14) = 16.999$ kJ

Power developed = Work done per cycle × no. of cycles per second

$= 16.999 \times 3 =$ **50.99 say 51 kW. (Ans.)**

☞**Example 13.26.** *In an engine working on Dual cycle, the temperature and pressure at the beginning of the cycle are 90°C and 1 bar respectively. The compression ratio is 9. The maximum pressure is limited to 68 bar and total heat supplied per kg of air is 1750 kJ. Determine :*

(*i*) *Pressure and temperatures at all salient points*

(*ii*) *Air standard efficiency*

(*iii*) *Mean effective pressure.*

Solution. Refer Fig. 13.22.

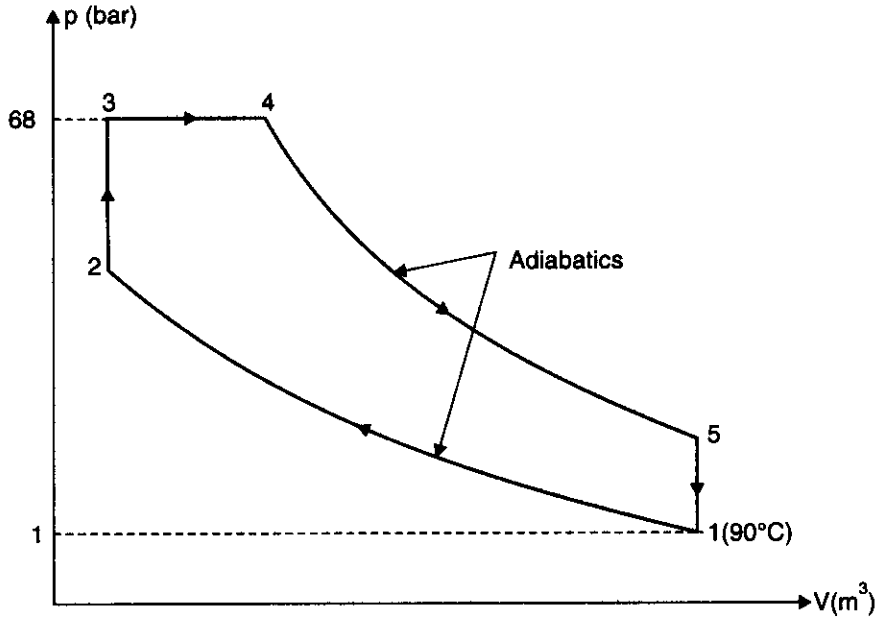


Fig. 13.22

Initial pressure, $p_1 = 1$ bar

Initial temperature, $T_1 = 90 + 273 = 363$ K

Compression ratio, $r = 9$

Maximum pressure, $p_3 = p_4 = 68$ bar

Total heat supplied $= 1750$ kJ/kg

(*i*) **Pressures and temperatures at salient points :**

For the *isentropic process 1-2,*

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

$$\boldsymbol{p_2} = p_1 \times \left(\frac{V_1}{V_2}\right)^{\gamma} = 1 \times (r)^{\gamma} = 1 \times (9)^{1.4} = \mathbf{21.67\ bar.}\quad \textbf{(Ans.)}$$

Also,
$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = (r)^{\gamma - 1} = (9)^{1.4 - 1} = 2.408$$

$\therefore$
$$T_2 = T_1 \times 2.408 = 363 \times 2.408 = \mathbf{874.1\ K.}\quad \textbf{(Ans.)}$$

$$\boldsymbol{p_3 = p_4} = \mathbf{68\ bar.}\quad \textbf{(Ans.)}$$

For the *constant volume process* 2-3,

$$\frac{p_2}{T_2} = \frac{p_3}{T_3}$$

$\therefore$ $$T_3 = T_2 \times \frac{p_3}{p_2} = 874.1 \times \frac{68}{21.67} = \mathbf{2742.9\ K.}\quad \textbf{(Ans.)}$$

Heat added at constant volume

$$= c_v\,(T_3 - T_2) = 0.71\,(2742.9 - 874.1) = 1326.8\ \text{kJ/kg}$$

$\therefore$ Heat added at constant pressure

= Total heat added – heat added at constant volume

$$= 1750 - 1326.8 = 423.2\ \text{kJ/kg}$$

$\therefore$ $$c_p(T_4 - T_3) = 423.2$$

or $$1.0(T_4 - 2742.9) = 423.2$$

$\therefore$ $$T_4 = \mathbf{3166\ K.}\quad \textbf{(Ans.)}$$

For *constant pressure process* 3-4,

$$\rho = \frac{V_4}{V_3} = \frac{T_4}{T_3} = \frac{3166}{2742.9} = 1.15$$

For *adiabatic (or isentropic) process* 4-5,

$$\frac{V_5}{V_4} = \frac{V_5}{V_2} \times \frac{V_2}{V_4} = \frac{V_1}{V_2} \times \frac{V_3}{V_4} = \frac{r}{\rho} \qquad \left(\because\ \rho = \frac{V_4}{V_3}\right)$$

Also $$p_4V_4^{\gamma} = p_5V_5^{\gamma}$$

$\therefore$ $$p_5 = p_4 \times \left(\frac{V_4}{V_5}\right)^{\gamma} = 68 \times \left(\frac{\rho}{r}\right)^{\gamma} = 68 \times \left(\frac{1.15}{9}\right)^{1.4} = \mathbf{3.81\ bar.}\quad \textbf{(Ans.)}$$

Again, $$\frac{T_5}{T_4} = \left(\frac{V_4}{V_5}\right)^{\gamma-1} = \left(\frac{\rho}{r}\right)^{\gamma-1} = \left(\frac{1.15}{9}\right)^{1.4-1} = 0.439$$

$\therefore$ $$T_5 = T_4 \times 0.439 = 3166 \times 0.439 = \mathbf{1389.8\ K.}\quad \textbf{(Ans.)}$$

(*ii*) **Air standard efficiency :**

Heat rejected during constant volume process 5-1,

$$Q_r = c_v(T_5 - T_1) = 0.71(1389.8 - 363) = 729\ \text{kJ/kg}$$

$\therefore$ $$\eta_{\text{air-standard}} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{Q_s - Q_r}{Q_s}$$

$$= \frac{1750 - 729}{1750} = \mathbf{0.5834\ or\ 58.34\%.}\quad \textbf{(Ans.)}$$

(*iii*) **Mean effective pressure, p_m :**

Mean effective pressure is given by

$$p_m = \frac{\text{Work done per cycle}}{\text{Stroke volume}}$$

or $$p_m = \frac{1}{V_s}\left[p_3\,(V_4 - V_3) + \frac{p_4V_4 - p_5V_5}{\gamma - 1} - \frac{p_2V_2 - p_1V_1}{\gamma - 1}\right]$$

$$V_1 = V_5 = r\,V_c,\ V_2 = V_3 = V_c,\ V_4 = \rho V_c,\quad V_s = (r-1)\,V_c \qquad \left[\because\ r = \frac{V_s + V_c}{V_c} = 1 + \frac{V_s}{V_c}\quad \therefore\ V_s = (r-1)\,V_c\right]$$

$$\therefore \quad p_m = \frac{1}{(r-1)V_c}\left[p_3(\rho V_c - V_c) + \frac{p_4\rho V_c - p_5 \times rV_c}{\gamma - 1} - \frac{p_2V_c - p_1rV_c}{\gamma - 1}\right]$$

$r = 9$, $\rho = 1.15$, $\gamma = 1.4$

$p_1 = 1$ bar, $p_2 = 21.67$ bar, $p_3 = p_4 = 68$ bar, $p_5 = 3.81$ bar

Substituting the above values in the above equation, we get

$$p_m = \frac{1}{(9-1)}\left[68(1.15-1) + \frac{68 \times 1.15 - 3.81 \times 9}{1.4 - 1} - \frac{21.67 - 9}{1.4 - 1}\right]$$

$$= \frac{1}{8}(10.2 + 109.77 - 31.67) = 11.04 \text{ bar}$$

Hence, *mean effective pressure* = **11.04 bar. (Ans.)**

Example 13.27. *An I.C. engine operating on the dual cycle (limited pressure cycle) the temperature of the working fluid (air) at the beginning of compression is 27°C. The ratio of the maximum and minimum pressures of the cycle is 70 and compression ratio is 15. The amounts of heat added at constant volume and at constant pressure are equal. Compute the air standard thermal efficiency of the cycle. State three main reasons why the actual thermal efficiency is different from the theoretical value.* **(U.P.S.C. 1997)**

Take γ for air = 1.4.

Solution. Refer Fig. 13.23. *Given :* $T_1 = 27 + 273 = 300$ K ; $\frac{p_3}{p_1} = 70$, $\frac{v_1}{v_2} = \frac{v_1}{v_3} = 15$

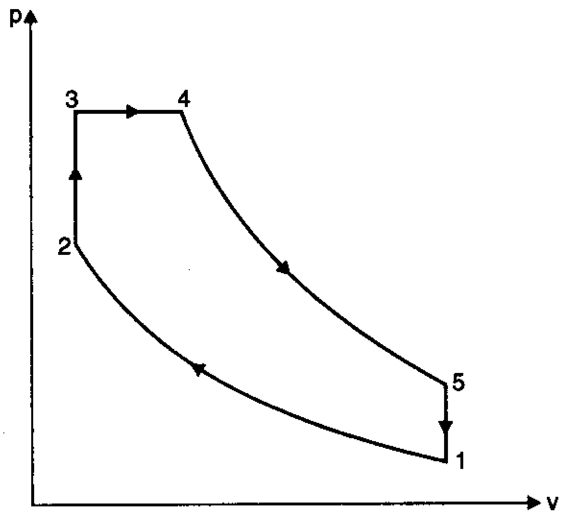


Fig. 13.23. Dual cycle.

Air standard efficiency, $\eta_{air\text{-}standard}$:

Consider 1 kg of air.

Adiabatic compression process 1-2 :

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = (15)^{1.4-1} = 2.954$$

$\therefore \quad T_2 = 300 \times 2.954 = 886.2 \text{ K}$

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = (15)^{1.4} \quad \Rightarrow \quad p_2 = 44.3\ p_1$$

Constant pressure process 2-3 :

$$\frac{p_2}{T_2} = \frac{p_3}{T_3}$$

or $$T_3 = T_2 \times \frac{p_3}{p_2} = 886.2 \times \frac{70p_1}{44.3p_1} = 1400 \text{ K}$$

Also, Heat added at constant volume = Heat added at constant pressure ...(Given)

or $$c_v\,(T_3 - T_2) = c_p\,(T_4 - T_3)$$

or $$T_3 - T_2 = \gamma\,(T_4 - T_3)$$

or $$T_4 = T_3 + \frac{T_3 - T_2}{\gamma} = 1400 + \frac{1400 - 886.2}{1.4} = 1767 \text{ K.}$$

Constant volume process 3-4 :

$$\frac{v_3}{T_3} = \frac{v_4}{T_4} \quad \Rightarrow \quad \frac{v_4}{v_3} = \frac{T_4}{T_3} = \frac{1767}{1400} = 1.26$$

Also, $$\frac{v_4}{v_3} = \frac{v_4}{(v_1/15)} = 1.26 \quad \text{or} \quad v_4 = 0.084\ v_1$$

Also, $$v_5 = v_1$$

Adiabatic expansion process 4-5 :

$$\frac{T_4}{T_5} = \left(\frac{v_5}{v_4}\right)^{\gamma - 1} = \left(\frac{v_1}{0.084 v_1}\right)^{1.4 - 1} = 2.69$$

$$\therefore \quad T_5 = \frac{T_4}{2.69} = \frac{1767}{2.69} = 656.9 \text{ K}$$

$$\therefore \quad \eta_{\text{air-standard}} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}}$$

$$= 1 - \frac{\text{Heat rejected}}{\text{Heat supplied}}$$

$$= 1 - \frac{c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)}$$

$$= 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

$$= 1 - \frac{(656.9 - 300)}{(1400 - 886.2) + 1.4(1767 - 1400)} = \mathbf{0.653} \text{ or } \mathbf{65.3\%}. \quad \textbf{(Ans.)}$$

Reasons for actual thermal efficiency being different from the theoretical value :

1. In theoretical cycle working substance is taken *air* whereas in actual cycle *air with fuel acts as working substance.*

2. The fuel combustion phenomenon and associated problems like dissociation of gases, dilution of charge during suction stroke, etc. have *not* been taken into account.

3. Effect of variable specific heat, heat loss through cylinder walls, inlet and exhaust velocities of air/gas etc. have *not* been taken into account.

☞**Example 13.28.** *A Diesel engine working on a dual combustion cycle has a stroke volume of 0.0085 m³ and a compression ratio 15 : 1. The fuel has a calorific value of 43890 kJ/kg. At the end of suction, the air is at 1 bar and 100°C. The maximum pressure in the cycle is 65 bar and air fuel ratio is 21 : 1. Find for ideal cycle the thermal efficiency. Assume* $c_p = 1.0$ *and* $c_v = 0.71$.

Solution. Refer Fig. 13.24.

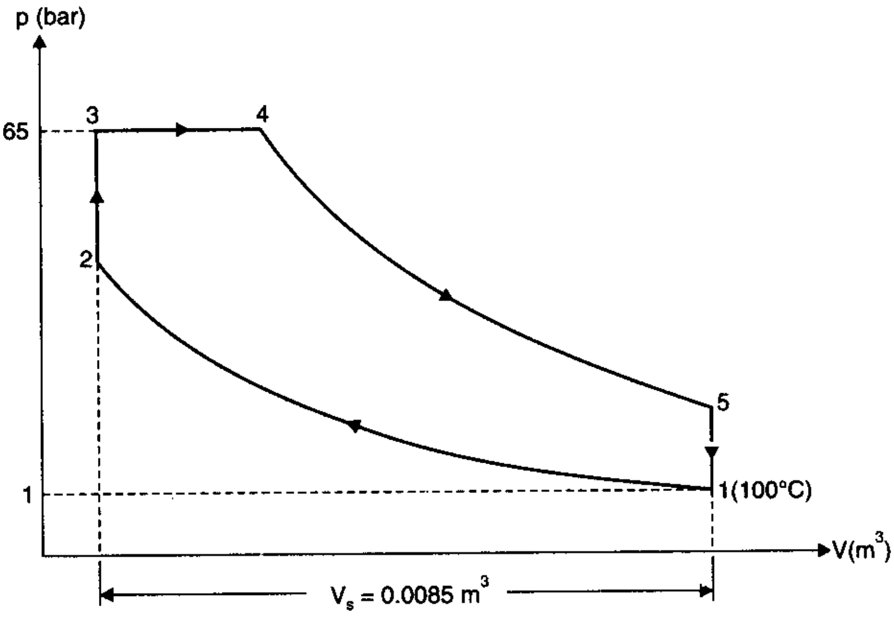


Fig. 13.24

Initial temperature, $T_1 = 100 + 273 = 373$ K

Initial pressure, $p_1 = 1$ bar

Maximum pressure in the cycle, $p_3 = p_4 = 65$ bar

Stroke volume, $V_s = 0.0085$ m³

Air-fuel ratio $= 21 : 1$

Compression ratio, $r = 15 : 1$

Calorific value of fuel, $C = 43890$ kJ/kg

$c_p = 1.0, \; c_v = 0.71$

Thermal efficiency :

$$V_s = V_1 - V_2 = 0.0085$$

and as $r = \dfrac{V_1}{V_2} = 15$, then $V_1 = 15V_2$

$\therefore$ $15V_2 - V_2 = 0.0085$

or $14V_2 = 0.0085$

or $V_2 = V_3 = V_c = \dfrac{0.0085}{14} = 0.0006 \text{ m}^3$

or $V_1 = 15V_2 = 15 \times 0.0006 = 0.009 \text{ m}^3$

For *adiabatic compression process 1-2,*

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

or $$p_2 = p_1 \cdot \left(\frac{V_1}{V_2}\right)^{\gamma} = 1 \times (15)^{1.41} \qquad \left[\gamma = \frac{c_p}{c_v} = \frac{1.0}{0.71} = 1.41\right]$$

$$= 45.5 \text{ bar}$$

Also, $$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r)^{\gamma-1} = (15)^{1.41-1} = 3.04$$

$\therefore$ $$T_2 = T_1 \times 3.04 = 373 \times 3.04 = 1134 \text{ K or } 861°\text{C}$$

For *constant volume process 2-3,*

$$\frac{p_2}{T_2} = \frac{p_3}{T_3}$$

or $$T_3 = T_2 \times \frac{p_3}{p_2} = 1134 \times \frac{65}{45.5} = 1620 \text{ K or } 1347°\text{C}$$

According to characteristic equation of gas,

$$p_1V_1 = mRT_1$$

$\therefore$ $$m = \frac{p_1V_1}{RT_1} = \frac{1 \times 10^5 \times 0.009}{287 \times 373} = 0.0084 \text{ kg (air)}$$

Heat added during constant volume process 2-3,

$$= m \times c_v (T_3 - T_2)$$
$$= 0.0084 \times 0.71 (1620 - 1134)$$
$$= 2.898 \text{ kJ}$$

Amount of fuel added during the constant volume process 2-3,

$$= \frac{2.898}{43890} = 0.000066 \text{ kg}$$

Also as air-fuel ratio is 21 : 1.

$\therefore$ Total amount of fuel added $$= \frac{0.0084}{21} = 0.0004 \text{ kg}$$

Quantity of fuel added during the process 3-4,

$$= 0.0004 - 0.000066 = 0.000334 \text{ kg}$$

$\therefore$ *Heat added during the constant pressure operation 3-4*

$$= 0.000334 \times 43890 = 14.66 \text{ kJ}$$

But $(0.0084 + 0.0004)\, c_p (T_4 - T_3) = 14.66$

or $0.0088 \times 1.0 (T_4 - 1620) = 14.66$

$\therefore$ $$T_4 = \frac{14.66}{0.0088} + 1620 = 3286 \text{ K or } 3013°\text{C}$$

Again for *process 3-4,*

$$\frac{V_3}{T_3} = \frac{V_4}{T_4} \quad \text{or} \quad V_4 = \frac{V_3T_4}{T_3} = \frac{0.0006 \times 3286}{1620} = 0.001217 \text{ m}^3$$

For *adiabatic expansion operation 4-5,*

$$\frac{T_4}{T_5} = \left(\frac{V_5}{V_4}\right)^{\gamma-1} = \left(\frac{0.009}{0.001217}\right)^{1.41-1} = 2.27$$

or $$T_5 = \frac{T_4}{2.27} = \frac{3286}{2.27} = 1447.5 \text{ K or } 1174.5°\text{C}$$

Heat rejected during constant volume process 5-1,

$= m\, c_v\, (T_5 - T_1)$

$= (0.00854 + 0.0004) \times 0.71\, (1447.5 - 373) = 6.713$ kJ

Work done = Heat supplied – Heat rejected

$= (2.898 + 14.66) - 6.713 = 10.845$ kJ

∴ *Thermal efficiency,*

$$\eta_{th.} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{10.845}{(2.898 + 14.66)} = \mathbf{0.6176 \text{ or } 61.76\%.} \quad \textbf{(Ans.)}$$

☞**Example 13.29.** *The compression ratio and expansion ratio of an oil engine working on the dual cycle are 9 and 5 respectively. The initial pressure and temperature of the air are 1 bar and 30°C. The heat liberated at constant pressure is twice the heat liberated at constant volume. The expansion and compression follow the law $pV^{1.25}$ = constant. Determine :*

(i) Pressures and temperatures at all salient points.

(ii) Mean effective pressure of the cycle.

(iii) Efficiency of the cycle.

(iv) Power of the engine if working cycles per second are 8.

Assume : Cylinder bore = 250 mm and stroke length = 400 mm.

Solution. Refer Fig. 13.25.

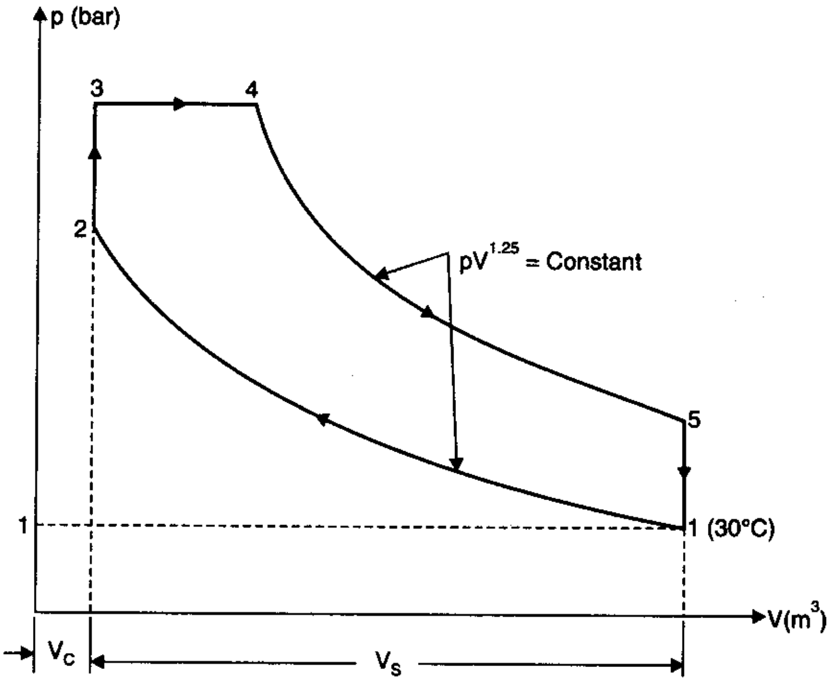


Fig. 13.25

Initial temperature, $T_1 = 30 + 273 = 303$ K

Initial pressure, $p_1 = 1$ bar

Compression and expansion law,

$$pV^{1.25} = \text{Constant}$$

Compression ratio, $r_c = 9$
Expansion ratio, $r_e = 5$
Number of cycles/sec. $= 8$
Cylinder diameter, $D = 250$ mm $= 0.25$ m
Stroke length, $L = 400$ mm $= 0.4$ m
Heat liberated at constant pressure
$= 2 \times$ heat liberated at constant volume

(*i*) **Pressure and temperatures at all salient points :**

For *compression process 1-2,*

$$p_1 V_1^n = p_2 V_2^n$$

$$\therefore \quad \boldsymbol{p_2} = p_1 \times \left(\frac{V_1}{V_2}\right)^n = 1 \times (9)^{1.25} = \mathbf{15.59\ bar.} \quad \textbf{(Ans.)}$$

Also,
$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} = (9)^{1.25-1} = 1.732$$

$$\therefore \quad \boldsymbol{T_2} = T_1 \times 1.732 = 303 \times 1.732$$
$$= \mathbf{524.8\ K\ or\ 251.8°C.} \quad \textbf{(Ans.)}$$

Also, $c_p(T_4 - T_3) = 2 \times c_v(T_3 - T_2)$ ...... (given) ...(*i*)

For *constant pressure process 3-4,*

$$\frac{T_4}{T_3} = \frac{V_4}{V_3} = \rho = \frac{\text{Compression ratio } (r_c)}{\text{Expansion ratio } (r_e)}$$
$$= \frac{9}{5} = 1.8$$
$$T_4 = 1.8 T_3$$

$$\left[\begin{aligned} \frac{V_5}{V_4}(i.e., r_e) &= \frac{V_5}{V_3} \times \frac{V_3}{V_4} \\ &= \frac{V_1}{V_3} \times \frac{1}{\rho} \\ &= \frac{V_1}{V_2} \times \frac{1}{\rho} = \frac{r_c}{\rho} \\ \therefore \ \rho &= \frac{r_c}{\frac{V_5}{V_4}} = \frac{r_c}{r_e} \end{aligned}\right]$$

Substituting the values of T_2 and T_4 in the eqn. (*i*), we get

$$1.0(1.8T_3 - T_3) = 2 \times 0.71(T_3 - 524.8)$$
$$0.8T_3 = 1.42(T_3 - 524.8)$$
$$0.8T_3 = 1.42T_3 - 745.2$$
$$\therefore \quad 0.62T_3 = 745.2$$
$$T_3 = \mathbf{1201.9\ K\ or\ 928.9°C.} \quad \textbf{(Ans.)}$$

Also,
$$\frac{p_3}{T_3} = \frac{p_2}{T_2} \qquad \text{...... for } \textit{process 2-3}$$

$$\therefore \quad \boldsymbol{p_3} = p_2 \times \frac{T_3}{T_2} = 15.59 \times \frac{1201.9}{524.8} = \mathbf{35.7\ bar.} \quad \textbf{(Ans.)}$$

$$\boldsymbol{p_4} = \boldsymbol{p_3} = \mathbf{35.7\ bar.} \quad \textbf{(Ans.)}$$

$$\boldsymbol{T_4} = 1.8T_3 = 1.8 \times 1201.9 = \mathbf{2163.4\ K\ or\ 1890.4°C.} \quad \textbf{(Ans.)}$$

For expansion process 4-5,

$$p_4 V_4^n = p_5 V_5^n$$

$$p_5 = p_4 \times \left(\frac{V_4}{V_5}\right)^n = p_4 \times \frac{1}{(r_e)^n} = \frac{35.7}{(5)^{1.25}} = \mathbf{4.77\ bar.}\ \ \textbf{(Ans.)}$$

Also
$$\frac{T_5}{T_4} = \left(\frac{V_4}{V_5}\right)^{n-1} = \frac{1}{(r_e)^{n-1}} = \frac{1}{(5)^{1.25-1}} = 0.668$$

$\therefore$ $T_5 = T_4 \times 0.668 = 2163.4 \times 0.668 =$ **1445 K or 1172°C. (Ans.)**

(*ii*) **Mean effective pressure, p_m :**

Mean effective pressure is given by

$$p_m = \frac{1}{V_s}\left[p_3(V_4 - V_3) + \frac{p_4V_4 - p_5V_5}{n-1} - \frac{p_2V_2 - p_1V_1}{n-1}\right]$$

$$= \frac{1}{(r_c - 1)}\left[p_3(\rho - 1) + \frac{p_4\rho - p_5r_c}{n-1} - \frac{p_2 - p_1r_c}{n-1}\right]$$

Now, $r_c = \rho$, $\rho = 1.8$, $n = 1.25$, $p_1 = 1$ bar, $p_2 = 15.59$ bar, $p_3 = 35.7$ bar, $p_4 = 35.7$ bar, $p_5 = 4.77$ bar

$\therefore$
$$p_m = \frac{1}{(9-1)}\left[35.7(1.8-1) + \frac{35.7 \times 1.8 - 4.77 \times 9}{1.25 - 1} - \frac{15.59 - 1 \times 9}{1.25 - 1}\right]$$

$$= \frac{1}{8}\,[28.56 + 85.32 - 26.36] = 10.94 \text{ bar}$$

Hence *mean effective pressure* = **10.94 bar. (Ans.)**

(*iii*) **Efficiency of the cycle :**

Work done per cycle is given by $W = p_mV_s$

Here, $V_s = \pi/4D^2L = \pi/4 \times 0.25^2 \times 0.4 = 0.0196$ m³

$\therefore$
$$W = \frac{10.94 \times 10^5 \times 0.0196}{1000} \text{ kJ/cycle} = 21.44 \text{ kJ/cycle}$$

Heat supplied per cycle $= mQ_s$,

where m is the mass of air per cycle which is given by

$$m = \frac{p_1V_1}{RT_1} \quad \text{where } V_1 = V_s + V_c = \frac{r_c}{r_c - 1}V_s$$

$$\left[\begin{array}{l} r_c = \dfrac{V_s + V_c}{V_c} = 1 + \dfrac{V_s}{V_c} \quad \text{or} \quad V_c = \dfrac{V_s}{r_c - 1} \\ \therefore\ V_1 = V_s + \dfrac{V_s}{r_c - 1} = V_s\left(1 + \dfrac{1}{r_c - 1}\right) = \dfrac{r_c}{r_c - 1}V_s \end{array}\right]$$

$$= \frac{9}{9-1} \times 0.0196 = 0.02205 \text{ m}^3$$

$\therefore$
$$m = \frac{1 \times 10^5 \times 0.02205}{287 \times 303} = 0.02535 \text{ kg/cycle}$$

$\therefore$ Heat supplied per cycle

$= mQ_s = 0.02535[c_v(T_3 - T_2) + c_p(T_4 - T_3)]$

$= 0.02535[0.71(1201.9 - 524.8) + 1.0(2163.4 - 1201.9)]$

$= 36.56$ kJ/cycle

$$\textit{Efficiency} = \frac{\text{Work done per cycle}}{\text{Heat supplied per cycle}} = \frac{21.44}{36.56}$$

= **0.5864 or 58.64%. (Ans.)**

(*iv*) **Power of the engine, P :**

Power of the engine, P = Work done per second

= Work done per cycle × no. of cycles/sec.

= 21.44 × 8 = **171.52 kW. (Ans.)**

13.7. COMPARISON OF OTTO, DIESEL AND DUAL COMBUSTION CYCLES

Following are the *important variable factors which are used as a basis for comparison of the cycles :*

- Compression ratio.
- Maximum pressure
- Heat supplied
- Heat rejected
- Net work

Some of the above mentioned variables are fixed when the performance of Otto, Diesel and dual combustion cycles is to be compared.

13.7.1. Efficiency Versus Compression Ratio

Fig. 13.26 shows the comparison for the air standard efficiencies of the Otto, Diesel and Dual combustion cycles at various compression ratios and with given cut-off ratio for the Diesel and Dual combustion cycles. It is evident from the Fig. 13.26 that the air standard efficiencies *increase with the increase in the compression ratio. For a given compression ratio Otto cycle is the most efficient while the Diesel cycle is the least efficient.* ($\eta_{otto} > \eta_{dual} > \eta_{diesel}$).

Note. The maximum compression ratio for the petrol engine is limited by detonation. In their respective ratio ranges, the Diesel cycle is more efficient than the Otto cycle.

13.7.2. For the Same Compression Ratio and the Same Heat Input

A comparison of the cycles (Otto, Diesel and Dual) on the p-v and T-s diagrams for the *same compression ratio and heat supplied* is shown in the Fig. 13.27.

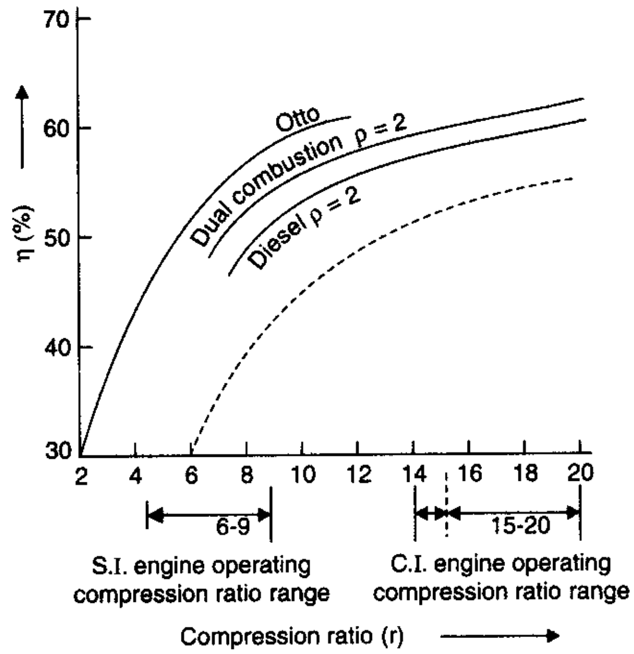


Fig. 13.26. Comparison of efficiency at various compression ratios.

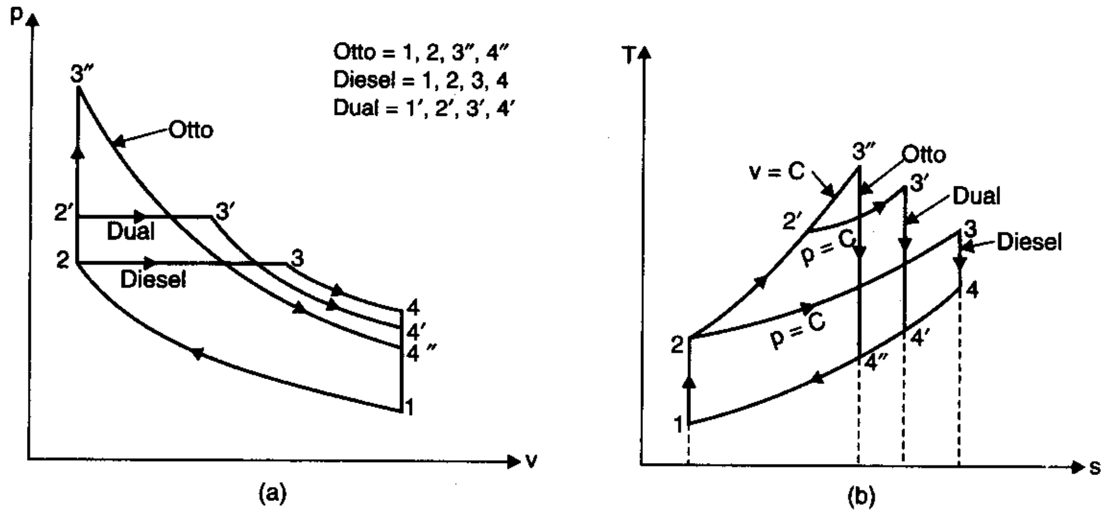


Fig. 13.27. (*a*) *p-v* diagram, (*b*) *T-s* diagram.

We know that, $$\eta = 1 - \frac{\text{Heat rejected}}{\text{Heat supplied}} \quad ...(13.13)$$

Since all the cycles reject their heat at the same specific volume, process line from state 4 to 1, the quantity of *heat rejected from each cycle is represented by the appropriate area under the line 4 to 1 on the T-s diagram*. As is evident from the eqn. (13.13) the cycle which has the least heat rejected will have the highest efficiency. Thus, Otto cycle is the most efficient and Diesel cycle is the least efficient of the three cycles.

i.e., $$\eta_{otto} > \eta_{dual} > \eta_{diesel}.$$

13.7.3. For Constant Maximum Pressure and Heat Supplied

Fig. 13.28 shows the Otto and Diesel cycles on *p-v* and *T-s* diagrams for constant maximum pressure and heat input respectively.

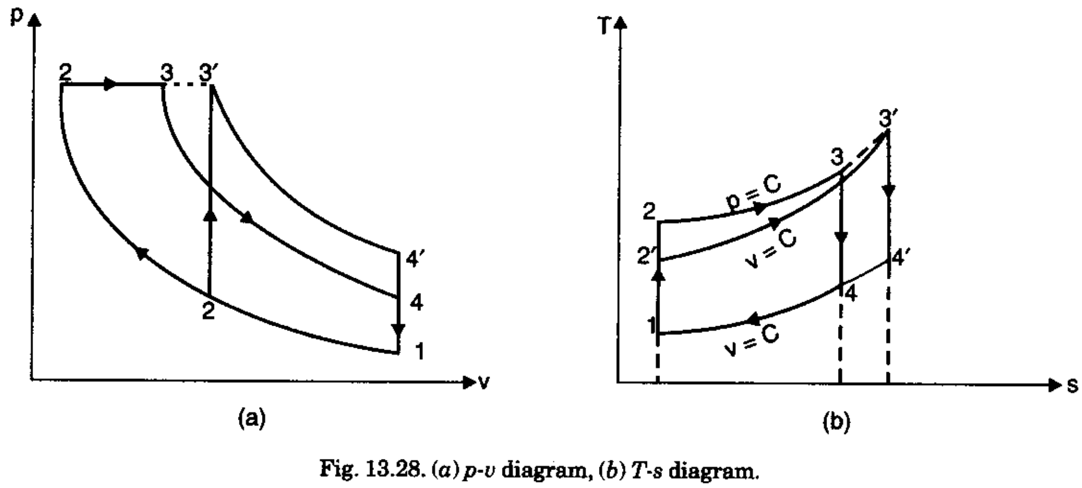


Fig. 13.28. (*a*) *p-v* diagram, (*b*) *T-s* diagram.

— For the maximum pressure the points 3 and 3′ must lie on a constant pressure line.

— On *T-s* diagram the heat rejected from the Diesel cycle is represented by the area under the line 4 to 1 and this area is less than the Otto cycle area under the curve 4′ to 1 ; hence the *Diesel cycle is more efficient than the Otto cycle for the condition of maximum pressure and heat supplied.*

Example 13.30. *With the help of p-v and T-s diagram compare the cold air standard otto, diesel and dual combustion cycles for same maximum pressure and maximum temperature.*

(AMIE Summer, 1998)

Solution. Refer Figs. 13.29 (*a*) and (*b*).

The air-standard Otto, Dual and Diesel cycles are drawn on common *p-v* and *T-s* diagrams for the same maximum pressure and maximum temperature, for the purpose of comparison.

Otto 1-2-3-4-1, Dual 1-2′-3′-3-4-1, Diesel 1-2″-3-4-1 (Fig 13.29 (*a*)).

Slope of constant volume lines on *T-s* diagram is higher than that of constant pressure lines. (Fig. 13.29 (*b*)).

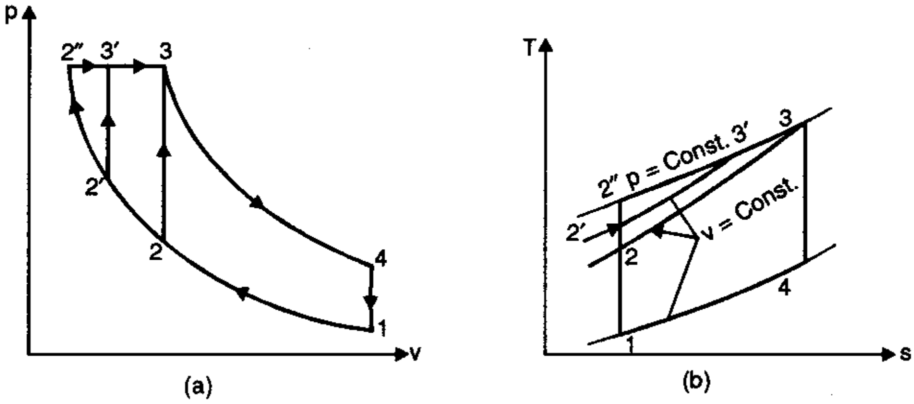


Fig. 13.29

Here the otto cycle must be limited to a low compression ratio (*r*) to fulfill the condition that point 3 (same maximum pressure and temperature) is to be a common state for all the three cycles.

The construction of cycles on *T-s* diagram proves that for the given conditions the heat rejected is same for all the three cycles (area under process line 4-1). Since, by definition,

$$\eta = 1 - \frac{\text{Heat rejected, } Q_r}{\text{Heat supplied, } Q_s} = 1 - \frac{\text{Const.}}{Q_s}$$

the cycle, with greater heat addition will be more efficient. From the *T-s* diagram,

$$Q_{s(\text{diesel})} = \text{Area under } 2''\text{-}3$$

$$Q_{s(\text{dual})} = \text{Area under } 2'\text{-}3'\text{-}3$$

$$Q_{s(\text{otto})} = \text{Area under } 2\text{-}3.$$

It can be seen that, $Q_{s(\text{diesel})} > Q_{s(\text{dual})} > Q_{s(\text{otto})}$

and thus, $\eta_{\text{diesel}} > \eta_{\text{dual}} > \eta_{\text{otto}}$.

13.8. ATKINSON CYCLE

This cycle consists of two adiabatics, a constant volume and a constant pressure process. *p-V* diagram of this cycle is shown in Fig. 13.30. It consists of the following *four operations* :

(*i*) 1-2—*Heat rejection at constant pressure*
(*ii*) 2-3—*Adiabatic compression*
(*iii*) 3-4—*Addition of heat at constant volume*
(*iv*) 4-1—*Adiabatic expansion.*

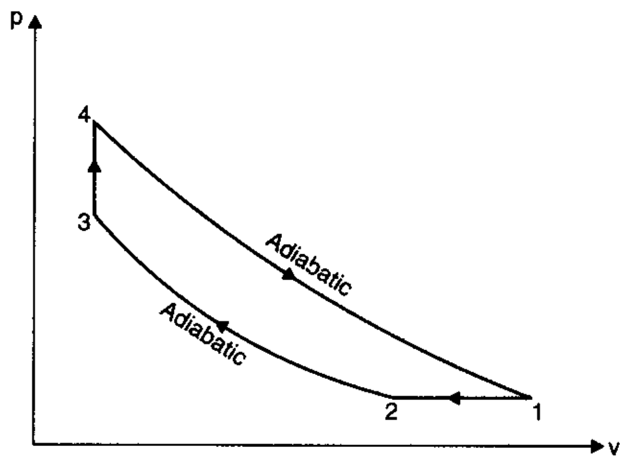


Fig. 13.30

Considering 1 kg of air.

Compression ratio $= \frac{v_2}{v_3} = \alpha$

Expansion ratio $= \frac{v_1}{v_4} = r$

Heat supplied at constant volume $= c_v(T_4 - T_3)$

Heat rejected $= c_v(T_1 - T_2)$

Work done = Heat supplied – heat rejected

$= c_v(T_4 - T_3) - c_v(T_1 - T_2)$

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{c_v\,(T_4 - T_3) - c_p(T_1 - T_2)}{c_v\,(T_4 - T_3)}$$

$$= 1 - \gamma \, . \, \frac{(T_1 - T_2)}{(T_4 - T_3)} \quad ...(i)$$

During *adiabatic compression 2-3,*

$$\frac{T_3}{T_2} = \left(\frac{v_2}{v_3}\right)^{\gamma - 1} = (\alpha)^{\gamma - 1}$$

or

$$T_3 = T_2 \ (\alpha)^{\gamma - 1} \quad ...(ii)$$

During *constant pressure operation 1-2,*

$$\frac{v_1}{T_1} = \frac{v_2}{T_2}$$

or
$$\frac{T_2}{T_1} = \frac{v_2}{v_1} = \frac{\alpha}{r} \quad ...(iii) \quad \left(\frac{v_2}{v_1} = \frac{v_2}{v_3} \times \frac{v_3}{v_1} = \frac{v_2}{v_3} \times \frac{v_4}{v_1} = \frac{\alpha}{r}\right)$$

During *adiabatic expansion 4-1,*

$$\frac{T_4}{T_1} = \left(\frac{v_1}{v_4}\right)^{\gamma - 1} = (r)^{\gamma - 1}$$

$$T_1 = \frac{T_4}{(r)^{\gamma - 1}} \quad ...(iv)$$

Putting the value of T_1 in eqn. (*iii*), we get

$$T_2 = \frac{T_4}{(r)^{\gamma - 1}} \cdot \frac{\alpha}{r}$$

$$= \frac{\alpha T_4}{r^{\gamma}} \quad ...(v)$$

Substituting the value of T_2 in eqn. (*ii*), we get

$$T_3 = \frac{\alpha T_4}{r^{\gamma}} (\alpha)^{\gamma - 1} = \left(\frac{\alpha}{r}\right)^{\gamma} . T_4$$

Finally putting the values of T_1, T_2 and T_3 in eqn. (*i*), we get

$$\eta = 1 - \gamma \left(\frac{\dfrac{T_4}{r^{\gamma - 1}} - \dfrac{\alpha T_4}{(r)^{\gamma}}}{T_4 - \left(\dfrac{\alpha}{r}\right)^{\gamma} . T_4}\right) = 1 - \gamma \left(\frac{r - \alpha}{r^{\gamma} - \alpha^{\gamma}}\right)$$

Hence, air standard efficiency $= 1 - \gamma . \left(\dfrac{r - \alpha}{r^{\gamma} - \alpha^{\gamma}}\right)$ ...(13.14)

Example 13.31. *A perfect gas undergoes a cycle which consists of the following processes taken in order :*

(*a*) *Heat rejection at constant pressure.*

(*b*) *Adiabatic compression from 1 bar and 27°C to 4 bar.*

(*c*) *Heat addition at constant volume to a final pressure of 16 bar.*

(*d*) *Adiabatic expansion to 1 bar.*

Calculate : (*i*) *Work done/kg of gas.*

(*ii*) *Efficiency of the cycle.*

Take : $c_p = 0.92$, $c_v = 0.75$.

Solution. Refer Fig. 13.31.

Pressure, $p_2 = p_1 = 1$ bar

Temperature, $T_2 = 27 + 273 = 300$ K

Pressure after adiabatic compression, $p_3 = 4$ bar

Final pressure after heat addition, $p_4 = 16$ bar

For *adiabatic compression 2-3,*

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma - 1}{\gamma}} = \left(\frac{4}{1}\right)^{\frac{1.22 - 1}{1.22}} = 1.284 \quad \left[\gamma = \frac{c_p}{c_v} = \frac{0.92}{0.75} = 1.22\right]$$

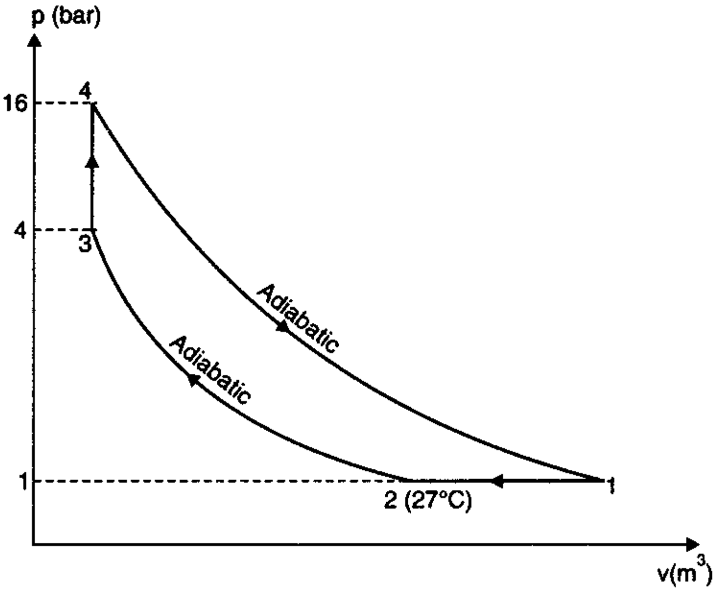


Fig. 13.31

$\therefore \quad T_3 = T_2 \times 1.284 = 300 \times 1.284 = 385.2$ K or 112.2°C

For *constant volume process 3-4,*

$$\frac{p_4}{T_4} = \frac{p_3}{T_3}$$

$$T_4 = \frac{p_4 T_3}{p_3} = \frac{16 \times 385.2}{4} = 1540.8 \text{ K or } 1267.8°\text{C}$$

For *adiabatic expansion process 4-1,*

$$\frac{T_4}{T_1} = \left(\frac{p_4}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} = \left(\frac{16}{1}\right)^{\frac{1.22 - 1}{1.22}} = 1.648$$

or

$$T_1 = \frac{T_4}{1.648} = \frac{1540.8}{1.648} = 934.9 \text{ K or } 661.9°\text{C}.$$

(*i*) **Work done per kg of gas, W** :

Heat supplied $= c_v (T_4 - T_3)$

$= 0.75 (1540.8 - 385.2) = 866.7$ kJ/kg

Heat rejected $= c_p (T_1 - T_2) = 0.92(934.9 - 300) = 584.1$ kJ/kg

Work done/kg of gas, W = Heat supplied – heat rejected

$= 866.7 - 584.1 = 282.6$ kJ/kg = **282600 N-m/kg. (Ans.)**

(*ii*) **Efficiency of the cycle :**

Efficiency, $$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{282.6}{866.7} = \mathbf{0.326 \text{ or } 32.6\%.} \quad \textbf{(Ans.)}$$

13.9. ERICSSON CYCLE

It is so named as it was invented by Ericsson. Fig. 13.32 shows *p-v* diagram of this cycle.

It comprises of the following *operations* :

(*i*) 1-2—*Rejection of heat at constant pressure*
(*ii*) 2-3—*Isothermal compression*
(*iii*) 3-4—*Addition of heat at constant pressure*
(*iv*) 4-1—*Isothermal expansion.*

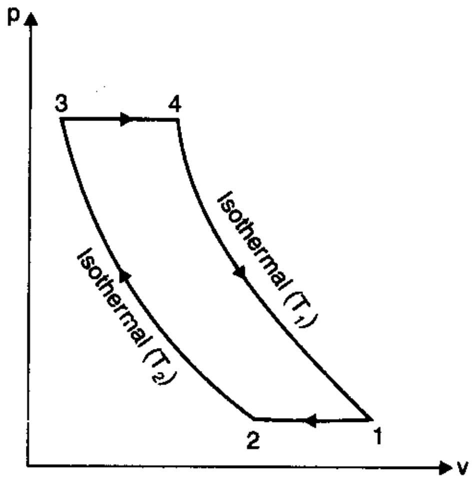


Fig. 13.32

Considering 1 kg of air.

Volume ratio, $r = \frac{v_2}{v_3} = \frac{v_1}{v_4}$

Heat supplied to air from an external source

= Heat supplied during the isothermal expansion 4-1

$= RT_1 \log_e r$

Heated rejected by air to an external source $= RT_2 \cdot \log_e r$

Work done = Heat supplied – heat rejected

$= RT_1 \cdot \log_e r - RT_2 \cdot \log_e r = R \log_e r\,(T_1 - T_2)$

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{R \log_e r\,(T_1 - T_2)}{RT_1 \cdot \log_e r}$$

$$= \frac{T_1 - T_2}{T_1} \quad \text{...(13.15)}$$

which is the same as Carnot cycle.

Note. For **'Stirling cycle', Miller cycle and Lenoir cycle** please refer to the Author's popular book on "I.C. Engines".

13.10. GAS TURBINE CYCLE—BRAYTON CYCLE

13.10.1. Ideal Brayton Cycle

Brayton cycle is *a constant pressure cycle for a perfect gas*. It is also called **Joule cycle.** The heat transfers are achieved in reversible constant pressure heat exchangers. An ideal gas turbine plant would perform the processes that make up a Brayton cycle. The cycle is shown in the Fig. 13.33 (*a*) and it is represented on *p-v* and *T-s* diagrams as shown in Figs. 13.33 (*b*) and (*c*).

The various operations are as follows :

Operation 1-2. The air is compressed isentropically from the lower pressure p_1 to the upper pressure p_2, the temperature rising from T_1 to T_2. No heat flow occurs.

Operation 2-3. Heat flows into the system increasing the volume from V_2 to V_3 and temperature from T_2 to T_3 whilst the pressure remains constant at p_2. Heat received = $mc_p (T_3 - T_2)$.

Operation 3-4. The air is expanded isentropically from p_2 to p_1, the temperature falling from T_3 to T_4. No heat flow occurs.

Operation 4-1. Heat is rejected from the system as the volume decreases from V_4 to V_1 and the temperature from T_4 to T_1 whilst the pressure remains constant at p_1. Heat rejected = $mc_p (T_4 - T_1)$.

$$\eta_{\text{air-standard}} = \frac{\text{Work done}}{\text{Heat received}}$$

$$= \frac{\text{Heat received/cycle} - \text{Heat rejected/cycle}}{\text{Heat received/cycle}}$$

$$= \frac{mc_p (T_3 - T_2) - mc_p (T_4 - T_1)}{mc_p (T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

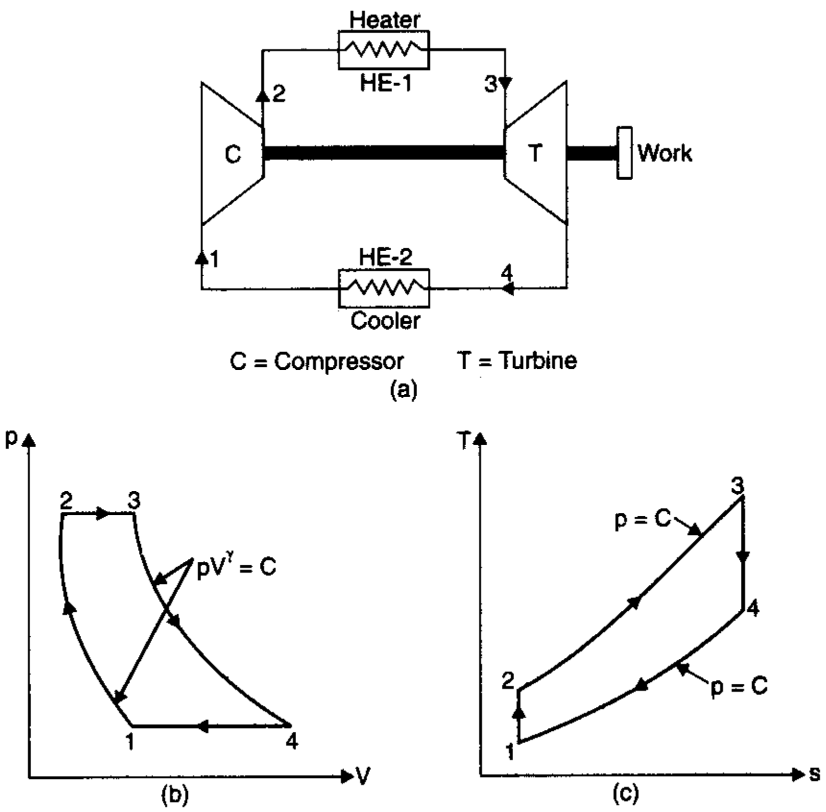


Fig. 13.33. Brayton cycle : (*a*) Basic components of a gas turbine power plant (*b*) *p-V* diagram (*c*) *T-s* diagram.

Now, from isentropic expansion,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = T_1 \, (r_p)^{\frac{\gamma-1}{\gamma}} \text{ , where } r_p = \text{pressure ratio.}$$

Similarly $$\frac{T_3}{T_4} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{or} \quad T_3 = T_4 \, (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \quad \eta_{\text{air-standard}} = 1 - \frac{T_4 - T_1}{T_4(r_p)^{\frac{\gamma-1}{\gamma}} - T_1(r_p)^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}} \quad ...(13.16)$$

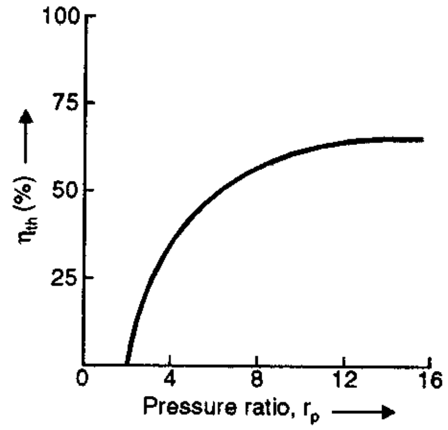


Fig. 13.34. Effect of pressure ratio on the efficiency of Brayton cycle.

The eqn. (13.16) shows that the *efficiency of the ideal joule cycle increases with the pressure ratio. The absolute limit of upper pressure is determined by the limiting temperature of the material of the turbine at the point at which this temperature is reached by the compression process alone, no further heating of the gas in the combustion chamber would be permissible and the work of expansion would ideally just balance the work of compression so that no excess work would be available for external use.*

13.10.2. Pressure Ratio for Maximum Work

Now we shall prove that the *pressure ratio for maximum work is a function of the limiting temperature ratio.*

Work output during the cycle

$$= \text{Heat received/cycle} - \text{heat rejected/cycle}$$

$$= mc_p \, (T_3 - T_2) - mc_p \, (T_4 - T_1)$$

$$= mc_p \, (T_3 - T_4) - mc_p \, (T_2 - T_1)$$

$$= mc_p \, T_3 \left(1 - \frac{T_4}{T_3}\right) - T_1 \left(\frac{T_2}{T_1} - 1\right)$$

In case of a given turbine the minimum temperature T_1 and the maximum temperature T_3 are prescribed, T_1 being the temperature of the atmosphere and T_3 the maximum temperature which the metals of turbine would withstand. Consider the specific heat at constant pressure c_p to be constant. Then,

Since, $$\frac{T_3}{T_4} = (r_p)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_1}$$

Using the constant $$'z' = \frac{\gamma-1}{\gamma},$$

we have, work output/cycle

$$W = K\left[T_3\left(1 - \frac{1}{r_p^{\,z}}\right) - T_1(r_p^{\,z} - 1)\right]$$

Differentiating with respect to r_p

$$\frac{dW}{dr_p} = K\left[T_3 \times \frac{z}{r_p(z+1)} - T_1 z r_p^{\,(z-1)}\right] = 0 \text{ for a maximum}$$

$$\therefore \quad \frac{zT_3}{r_p^{\,(z+1)}} = T_1 z (r_p)^{(z-1)}$$

$$\therefore \quad r_p^{\,2z} = \frac{T_3}{T_1}$$

$$\therefore \quad r_p = (T_3/T_1)^{1/2z} \quad i.e., \quad r_p = (T_3/T_1)^{\frac{\gamma}{2(\gamma-1)}} \qquad ...(13.17)$$

Thus, the *pressure ratio for maximum work is a function of the limiting temperature ratio.*

13.10.3. Work Ratio

Work ratio is defined as the *ratio of net work output to the work done by the turbine.*

$$\therefore \quad \text{Work ratio} = \frac{W_T - W_C}{W_T}$$

[where, W_T = Work obtained from this turbine,
and W_C = Work supplied to the compressor.]

$$= \frac{mc_p(T_3 - T_4) - mc_p(T_2 - T_1)}{mc_p(T_3 - T_4)} = 1 - \frac{T_2 - T_1}{T_3 - T_4}$$

$$= 1 - \frac{T_1}{T_3}\left[\frac{(r_p)^{\frac{\gamma-1}{\gamma}} - 1}{1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}}\right] = 1 - \frac{T_1}{T_3}(r_p)^{\frac{\gamma-1}{\gamma}} \qquad ...(13.18)$$

13.10.4. Open Cycle Gas Turbine—Actual Brayton Cycle

Refer Fig. 13.35. The fundamental gas turbine unit is one operating on the open cycle in which a rotary compressor and a turbine are mounted on a common shaft. Air is drawn into the compressor and after compression passes to a combustion chamber. Energy is supplied in the combustion chamber by spraying fuel into the air stream, and the resulting hot gases expand through the turbine to the atmosphere. In order to achieve net work output from the unit, the turbine must develop more gross work output than is required to drive the compressor and to overcome mechanical losses in the drive. The products of combustion coming out from the turbine are exhausted to the atmosphere as they cannot be used any more. The working fluids (air and fuel) must be replaced continuously as they are exhausted into the atmosphere.

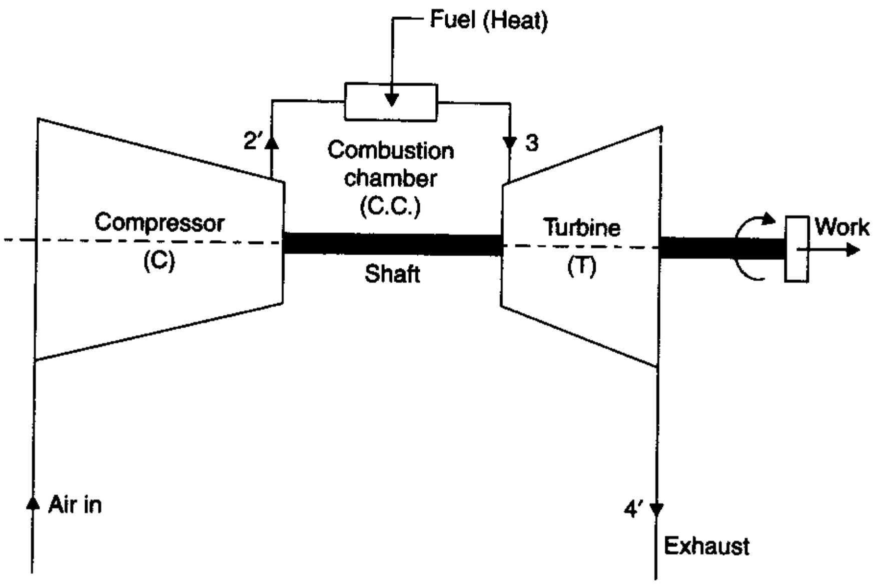


Fig. 13.35. Open cycle gas turbine.

If pressure loss in the combustion chamber is neglected, this cycle may be drawn on a T-s diagram as shown in Fig. 13.36.

- 1-2′ represents : *irreversible adiabatic compression.*
- 2′-3 represents : *constant pressure heat supply in the combustion chamber.*
- 3-4′ represents : *irreversible adiabatic expansion.*
- 1-2 represents : *ideal isentropic compression.*
- 3-4 represents : *ideal isentropic expansion.*

Assuming change in kinetic energy between the various points in the cycle to be negligibly small compared with enthalpy changes and then applying the flow equation to each part of cycle, for unit mass, we have

Work input (compressor) $= c_p (T_2' - T_1)$

Heat supplied (combustion chamber) $= c_p (T_3 - T_2')$

Work output (turbine) $= c_p (T_3 - T_4')$

$\therefore$ Net work output = Work output − Work input

$= c_p (T_3 - T_4') - c_p(T_2' - T_1)$

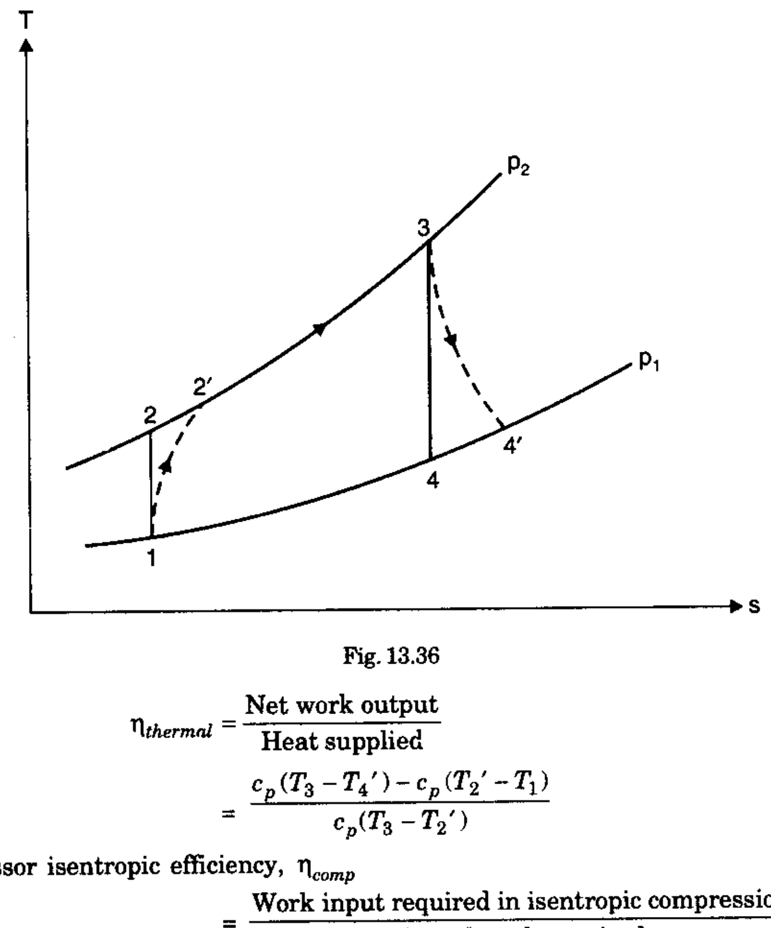


Fig. 13.36

and
$$\eta_{thermal} = \frac{\text{Net work output}}{\text{Heat supplied}}$$
$$= \frac{c_p(T_3 - T_4') - c_p(T_2' - T_1)}{c_p(T_3 - T_2')}$$

Compressor isentropic efficiency, η_{comp}
$$= \frac{\text{Work input required in isentropic compression}}{\text{Actual work required}}$$
$$= \frac{c_p(T_2 - T_1)}{c_p(T_2' - T_1)} = \frac{T_2 - T_1}{T_2' - T_1} \quad \text{...(13.19)}$$

Turbine isentropic efficiency, $\eta_{turbine}$
$$= \frac{\text{Actual work output}}{\text{Isentropic work output}}$$
$$= \frac{c_p(T_3 - T_4')}{c_p(T_3 - T_4)} = \frac{T_3 - T_4'}{T_3 - T_4} \quad \text{...(13.20)}$$

Note. With the variation in temperature, the value of the specific heat of a real gas varies, and also in the open cycle, the specific heat of the gases in the combustion chamber and in turbine is different from that in the compressor because fuel has been added and a chemical change has taken place. Curves showing the variation of c_p with temperature and air/fuel ratio can be used, and a suitable mean value of c_p and hence γ can be found out. It is usual in gas turbine practice to assume fixed mean value of c_p and γ for the expansion process, and fixed mean values of c_p and γ for the compression process. In an open cycle gas turbine unit the mass flow of gases in turbine is greater than that in compressor due to mass of fuel burned, but it is possible to neglect mass of fuel, since the air/fuel ratios used are large. Also, in many cases, air is bled from the compressor for cooling purposes, or in the case of air-craft at high altitudes, bled air is used for de-icing and cabin air-conditioning. This amount of air bled is approximately the same as the mass of fuel injected therein.

13.10.5. Methods for Improvement of Thermal Efficiency of Open Cycle Gas Turbine Plant

The following methods are employed *to increase the specific output and thermal efficiency of the plant :*

1. Intercooling 2. Reheating 3. Regeneration.

1. **Intercooling.** A compressor in a gas turbine cycle utilises the major percentage of power developed by the gas turbine. The work required by the compressor can be reduced by compressing the air in two stages and *incorporating an intercooler* between the two as shown in Fig. 13.37. The corresponding *T-s* diagram for the unit is shown in Fig. 13.38. The actual processes take place as follows :

1-2′	...	L.P. (Low pressure) compression
2′-3	...	Intercooling
3-4′	...	H.P. (High pressure) compression
4′-5	...	C.C. (Combustion chamber)-heating
5-6′	...	T (Turbine)-expansion

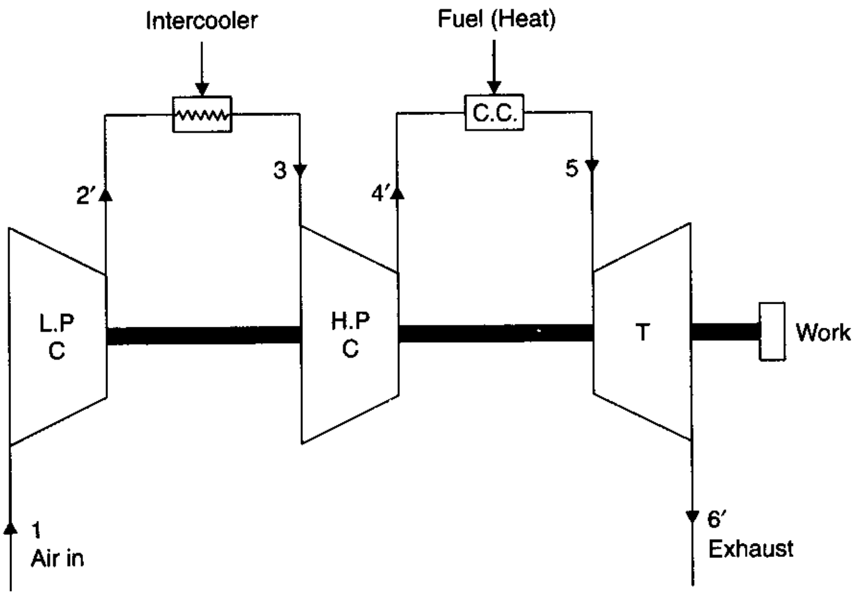


Fig. 13.37. Turbine plant with intercooler.

The ideal cycle for this arrangement is 1-2-3-4-5-6 ; the compression process without intercooling is shown as 1-*L*′ in the actual case, and 1-*L* in the ideal isentropic case.

Now,

Work input (*with intercooling*)

$$= c_p(T_2' - T_1) + c_p(T_4' - T_3) \quad ...(13.21)$$

Work input (*without intercooling*)

$$= c_p(T_L' - T_1) = c_p(T_2' - T_1) + c_p(T_L' - T_2') \quad ...(13.22)$$

By comparing equation (13.22) with equation (13.21) it can be observed that the *work input with intercooling is less than the work input with no intercooling*, when $c_p\ (T_4' - T_3)$ is *less* than $c_p(T_L' - T_2')$. This is so if it is assumed that isentropic efficiencies of the two compressors, operating separately, are each equal to the isentropic efficiency of the single compressor which would be required if no intercooling were used. Then $(T_4' - T_3) < (T_L' - T_2')$ since the pressure lines diverge on the *T-s* diagram from left to the right.

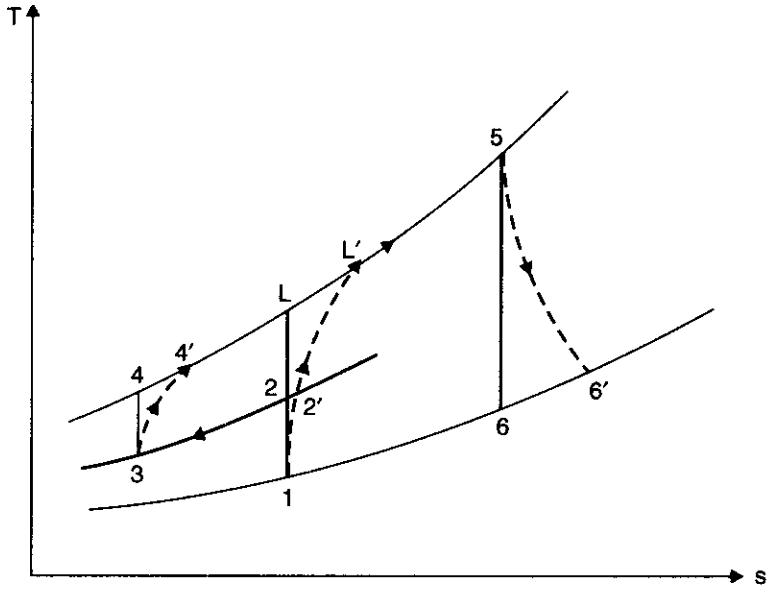


Fig. 13.38. *T-s* diagram for the unit.

Again, work ratio $= \dfrac{\text{Net work output}}{\text{Gross work output}}$

$$= \frac{\text{Work of expansion} - \text{Work of compression}}{\text{Work of expansion}}$$

From this we may conclude that *when the compressor work input is reduced then the work ratio is increased.*

However the heat supplied in the combustion chamber when intercooling is used in the cycle, is given by,

Heat supplied *with intercooling* $= c_p(T_5 - T_4')$

Also the heat supplied when intercooling is not used, with the same maximum cycle temperature T_5, is given by

Heat supplied *without intercooling* $= c_p\ (T_5 - T_L')$

Thus, the *heat supplied when intercooling is used is greater than with no intercooling. Although the net work output is increased by intercooling it is found in general that the increase in heat to be supplied causes the thermal efficiency to decrease.* When intercooling is used a supply of cooling water must be readily available. The additional bulk of the unit may offset the advantage to be gained by increasing the work ratio.

2. **Reheating.** The output of a gas turbine can be amply improved by expanding the gases in two stages with a *reheater* between the two as shown in Fig. 13.39. The H.P. turbine drives the compressor and the L.P. turbine provides the useful power output. The corresponding *T-s* diagram is shown in Fig. 13.40. The line *4'-L'* represents the expansion in the L.P. turbine if reheating is *not* employed.

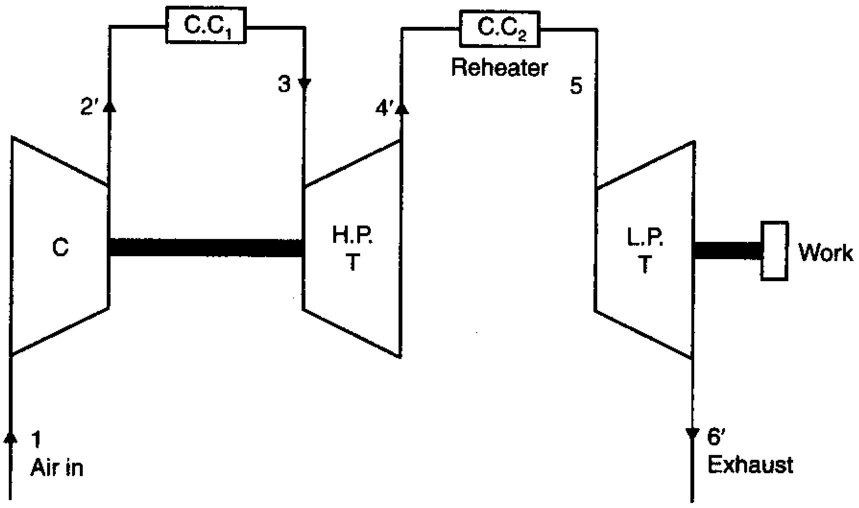


Fig. 13.39. Gas turbine with reheater.

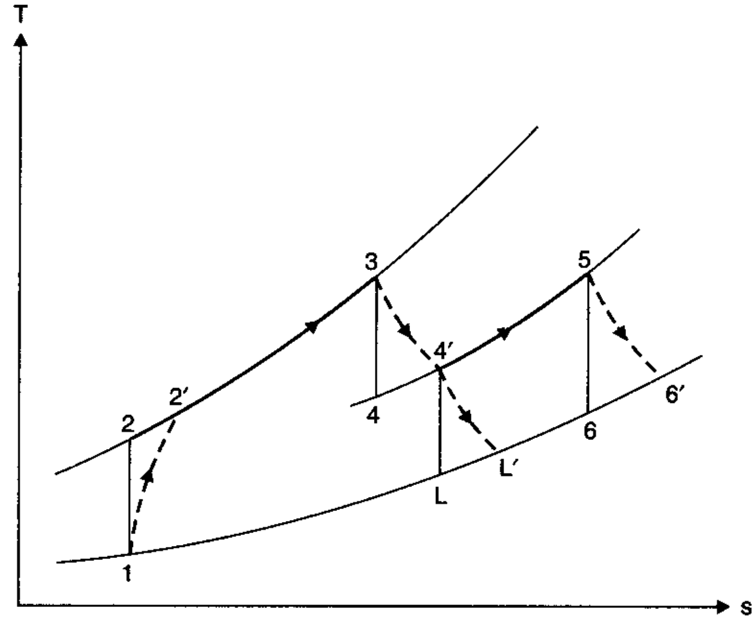


Fig. 13.40. *T-s* diagram for the unit.

Neglecting mechanical losses the work output of the H.P. turbine must be exactly equal to the work input required for the compressor i.e., $c_{pa}(T_2' - T_1) = c_{pg}(T_3 - T_4')$

The work output (net output) of L.P. turbine is given by,

Net work output (with reheating) $= c_{pg}(T_5 - T_6')$

and Net work output (without reheating) $= c_{pg}(T_4' - T_L')$

Since the pressure lines diverge to the right on *T-s* diagram it can be seen that the temperature difference $(T_5 - T_6')$ is always *greater* than $(T_4' - T_L')$, so that *reheating increases the net work output.*

Although net work is increased by reheating the heat to be supplied is also increased, and the net effect *can be to reduce the thermal efficiency*

Heat supplied $= c_{pg} (T_3 - T_2') + c_{pg} (T_5 - T_4')$.

Note. c_{pa} and c_{pg} stand for specific heats of air and gas respectively at constant pressure.

3. **Regeneration.** The exhaust gases from a gas turbine carry a large quantity of heat with them since their temperature is far above the ambient temperature. They can be used to heat the air coming from the compressor thereby reducing the mass of fuel supplied in the combustion chamber. Fig. 13.41 shows a gas turbine plant with a regenerator. The corresponding *T-s* diagram is shown in Fig. 13.42. 2′-3 represents the heat flow into the compressed air during its passage through the heat exchanger and 3-4 represents the heat taken in from the combustion of fuel. Point 6 represents the temperature of exhaust gases at discharge from the heat exchanger. The maximum temperature to which the air could be heated in the heat exchanger is ideally that of exhaust gases, but less than this is obtained in practice because a temperature gradient must exist for an unassisted transfer of energy. The *effectiveness* of the heat exchanger is given by :

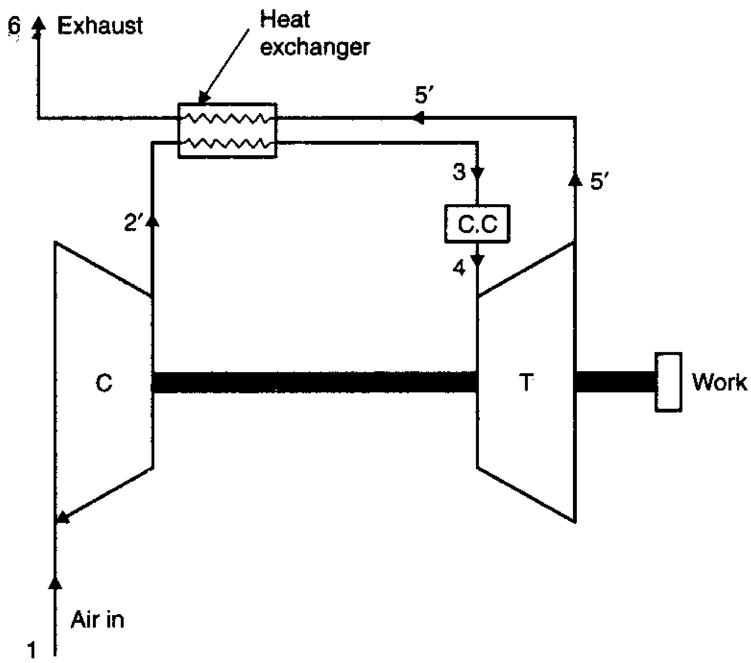


Fig. 13.41. Gas turbine with regenerator.

Effectiveness, $$\varepsilon = \frac{\text{Increase in enthalpy per kg of air}}{\text{Available increase in enthalpy per kg of air}}$$

$$= \frac{(T_3 - T_2')}{(T_5' - T_2')} \quad \text{...(13.23)}$$

(assuming c_{pa} and c_{pg} to be equal)

A heat exchanger is usually used in large gas turbine units for marine propulsion or industrial power.

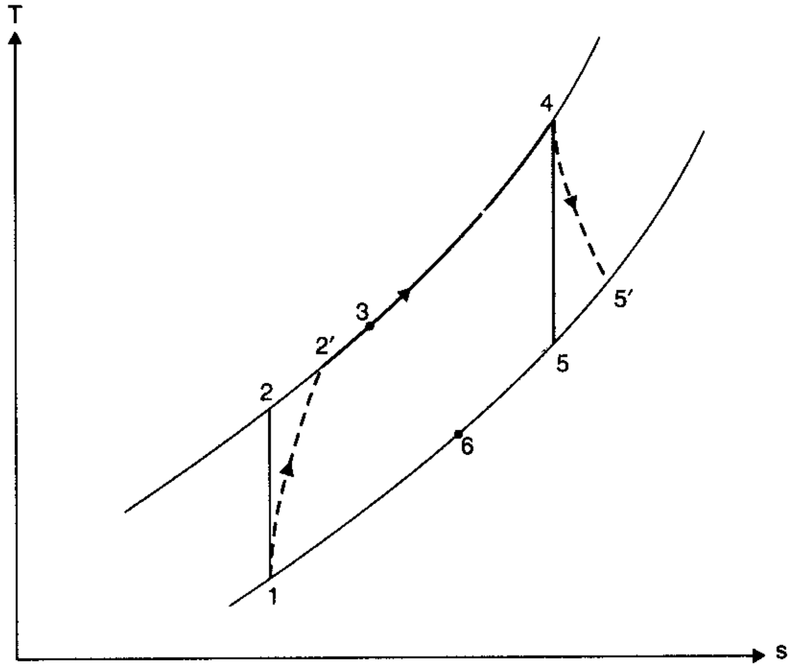


Fig. 13.42. *T-s* diagram for the unit.

13.10.6. Effect of Operating Variables on Thermal Efficiency

The thermal efficiency of *actual open cycle* depends on the following thermodynamic variables :

(*i*) Pressure ratio

(*ii*) Turbine inlet temperature (T_3)

(*iii*) Compressor inlet temperature (T_1)

(*iv*) Efficiency of the turbine ($\eta_{turbine}$)

(*v*) Efficiency of the compressor (η_{comp}).

Effect of turbine inlet temperature and pressure ratio :

If the permissible turbine inlet-temperature (with the other variables being constant) of an *open cycle gas turbine power plant* is increased its *thermal efficiency* is *amply improved.* A practical limitation to increasing the turbine inlet temperature, however, is the ability of the material available for the turbine blading to *withstand the high rotative and thermal stresses.*

Refer Fig. 13.43. For *a given turbine inlet temperature, as the pressure ratio increases, the heat supplied as well as the heat rejected are reduced. But the ratio of change of heat supplied is not the same as the ratio of change heat rejected.* As a consequence, there *exists an optimum pressure ratio producing maximum thermal efficiency for a given turbine inlet temperature.*

As the pressure ratio increases, the thermal efficiency also increases until it becomes maximum and then it drops off with a further increase in pressure ratio (Fig. 13.44). Further, as the turbine inlet temperature increases, the peaks of the curves flatten out giving a greater range of ratios of pressure optimum efficiency.

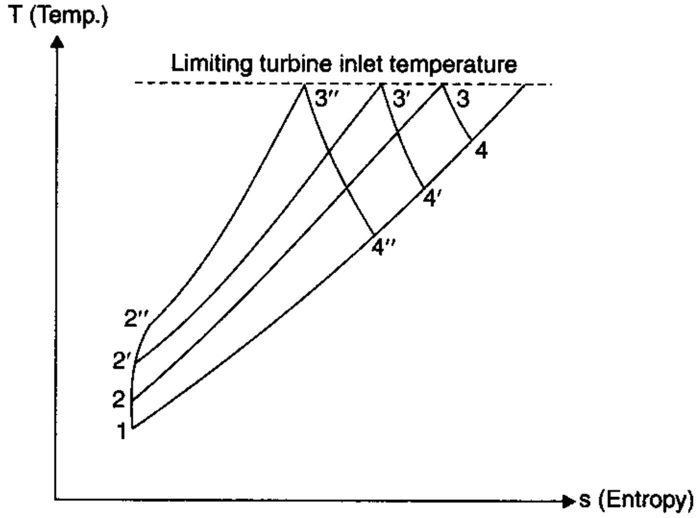


Fig. 13.43

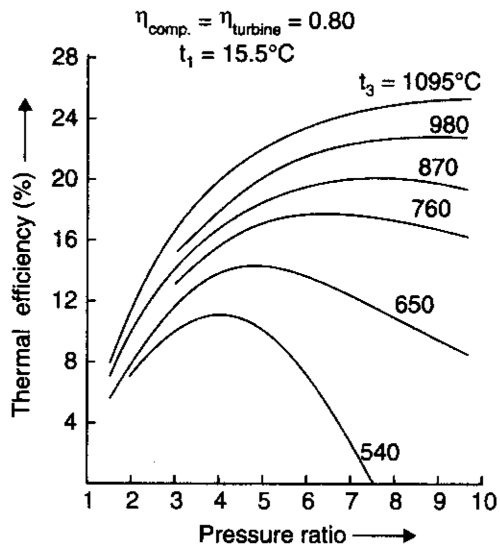


Fig. 13.44. Effect of pressure ratio and turbine inlet temperature.

Following particulars are worthnoting :

Gas temperatures	*Efficiency (gas turbine)*
550 to 600°C	20 to 22%
900 to 1000°C	32 to 35%
Above 1300°C	more than 50%

Effect of turbine and compressor efficiencies :

Refer Fig. 13.45. The thermal efficiency of the actual gas turbine cycle is very sensitive to variations in the efficiencies of the compressor and turbine. There is a particular pressure ratio at which maximum efficiencies occur. For lower efficiencies, the peak of the thermal efficiency occurs at lower pressure ratios and vice versa.

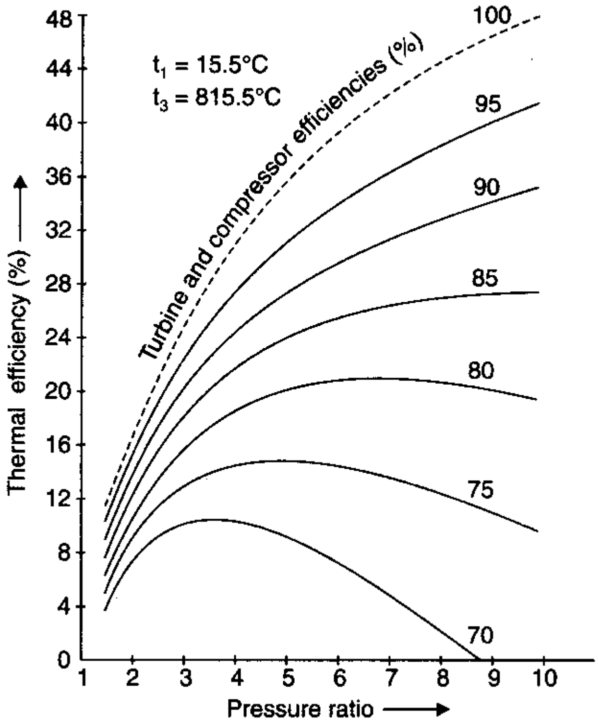


Fig. 13.45. Effect of component efficiency.

Effect of compressor inlet temperature :

Refer Fig. 13.46 (on next page). *With the decrease in the compressor inlet temperature there is increase in thermal efficiency of the plant.* Also the peaks of thermal efficiency occur at high pressure ratios and the curves become flatter giving thermal efficiency over a wider pressure ratio range.

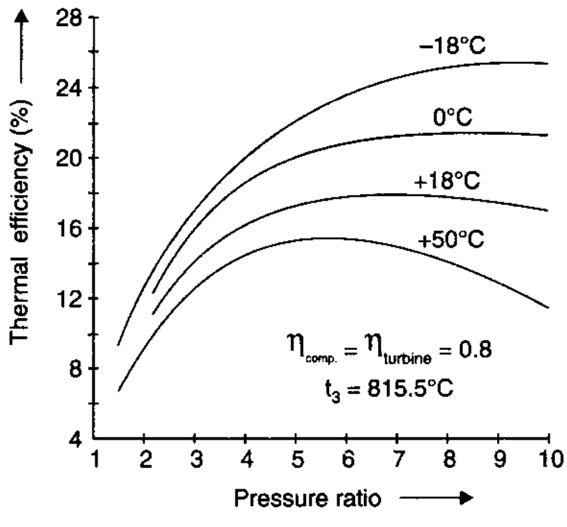


Fig. 13.46

13.10.7. Closed Cycle Gas Turbine (Constant pressure or joule cycle).

Fig. 13.47 shows a gas turbine operating on a constant pressure cycle in which the closed system consists of air behaving as an ideal gas. The various operations are as follows : Refer Figs. 13.48 and 13.49.

Operation 1-2 : The air is compressed isentropically from the lower pressure p_1 to the upper pressure p_2, the temperature rising from T_1 to T_2. No heat flow occurs.

Operation 2-3 : Heat flow into the system increasing the volume from V_2 to V_3 and temperature from T_2 to T_3 whilst the pressure remains constant at p_2. Heat received = $mc_p\ (T_3 - T_2)$.

Operation 3-4 : The air is expanded isentropically from p_2 to p_1, the temperature falling from T_3 to T_4. No heat flow occurs.

Operation 4-1 : Heat is rejected from the system as the volume decreases from V_4 to V_1 and the temperature from T_4 to T_1 whilst the pressure remains constant at p_1. Heat rejected = $mc_p\ (T_4 - T_1)$

$$\eta_{air\text{-}standard} = \frac{\text{Work done}}{\text{Heat received}}$$

$$= \frac{\text{Heat received/cycle} - \text{Heat rejected/cycle}}{\text{Heat received/cycle}}$$

$$= \frac{mc_p(T_3 - T_2) - mc_p(T_4 - T_1)}{mc_p(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

Now, from isentropic expansion

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

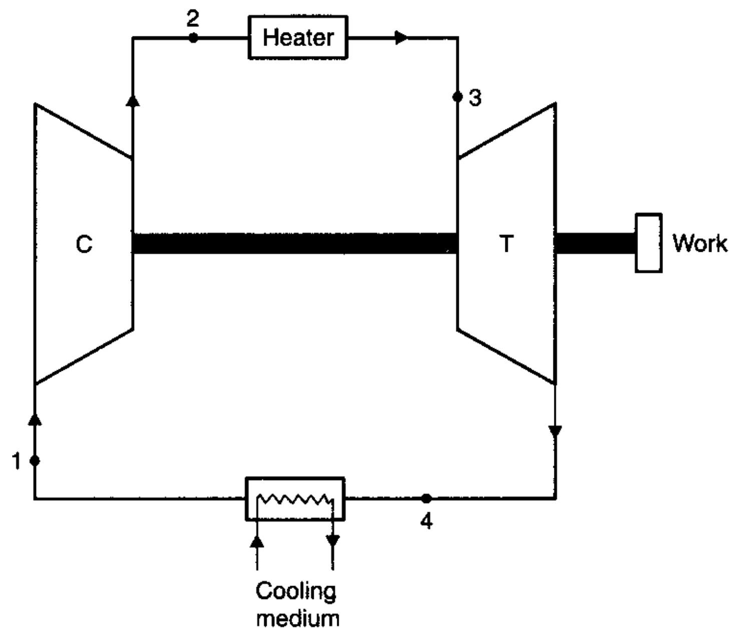


Fig. 13.47. Closed cycle gas turbine.

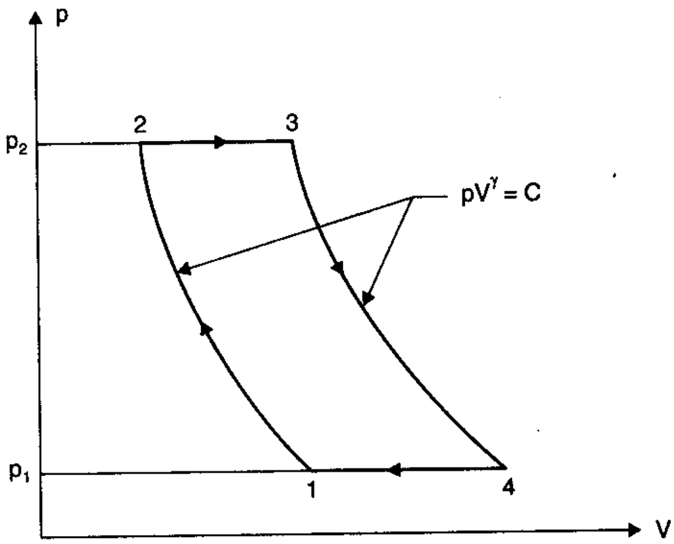


Fig. 13.48. *p-V* diagram.

$$T_2 = T_1 \, (r_p)^{\frac{\gamma-1}{\gamma}}, \text{ where } r_p = \text{Pressure ratio}$$

Similarly $$\frac{T_3}{T_4} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \text{ or } T_3 = T_4 \, (r_p)^{\frac{\gamma-1}{\gamma}}$$

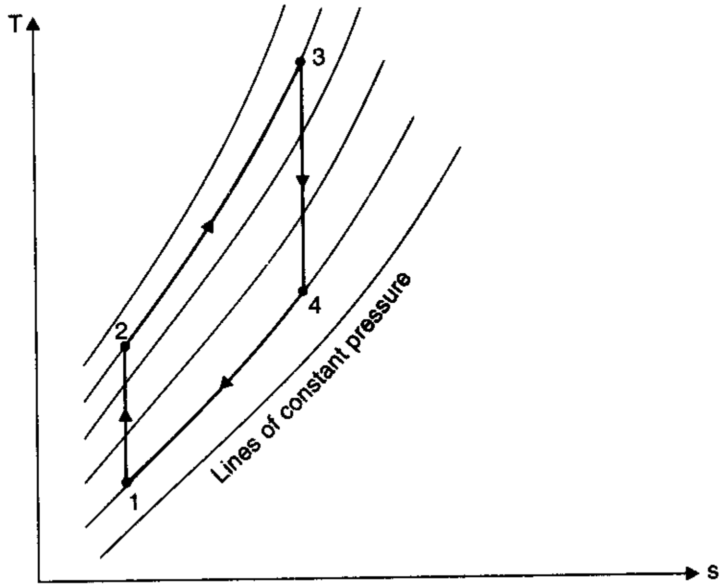


Fig. 13.49. *T-s* diagram.

$$\therefore \quad \eta_{air\text{-}standard} = 1 - \frac{T_4 - T_1}{T_4 (r_p)^{\frac{\gamma-1}{\gamma}} - T_1 (r_p)^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}} \quad ...(13.24)$$

The expression shows that the *efficiency of the ideal joule cycle increases with the pressure ratio. The absolute limit of pressure is determined by the limiting temperature of the material of the turbine at the point at which this temperature is reached by the compression process alone, no further heating of the gas in the combustion chamber would be permissible and the work of expansion would ideally just balance the work of compression so that no excess work would be available for external use.*

Now we shall prove that the *pressure ratio for maximum work is a function of the limiting temperature ratio.*

Work output during the cycle

$$= \text{Heat received/cycle} - \text{Heat rejected/cycle}$$

$$= mc_p (T_3 - T_2) - mc_p (T_4 - T_1) = mc_p (T_3 - T_4) - mc_p (T_2 - T_1)$$

$$= mc_p T_3 \left(1 - \frac{T_4}{T_3}\right) - T_1 \left(\frac{T_2}{T_1} - 1\right)$$

In case of a given turbine the minimum temperature T_1 and the maximum temperature T_3 are prescribed, T_1 being the temperature of the atmosphere and T_3 the maximum temperature which the metals of turbine would withstand. Consider the specific heat at constant pressure c_p to be constant. Then,

Since, $\quad \dfrac{T_3}{T_4} = (r_p)^{\frac{\gamma-1}{\gamma}} = \dfrac{T_2}{T_1}$

Using the constant $\quad$ 'z' $= \dfrac{\gamma - 1}{\gamma}$,

we have, work output/cycle $\quad W = K \left[T_3 \left(1 - \dfrac{1}{r_p^{\,z}}\right) - T_1 (r_p^{\,z} - 1) \right]$

Differentiating with respect to r_p

$$\frac{dW}{dr_p} = K \left[T_3 \times \frac{z}{r_p(z+1)} - T_1 z r_p^{(z-1)} \right] = 0 \text{ for a maximum}$$

$$\therefore \quad \frac{zT_3}{r_p^{(z+1)}} = T_1 z (r_p)^{(z-1)}$$

$$\therefore \quad r_p^{2z} = \frac{T_3}{T_1}$$

$$\therefore \quad r_p = (T_3/T_1)^{1/2z} \text{ i.e., } r_p = (T_3/T_1)^{\frac{\gamma}{2(\gamma-1)}}$$

Thus the *pressure ratio for maximum work is a function of the limiting temperature ratio.*

Fig. 13.50 shows an arrangement of closed cycle stationary gas turbine plant in which air is continuously circulated. This ensures that the air is not polluted by the addition of combustion waste product, since the heating of air is carried out in the form of heat exchanger shown in the diagram as air heater. The air exhausted from the power turbine is cooled before readmission to L.P. compressor. The various operations as indicated on T-s diagram (Fig. 13.51) are as follows :

Operation 1-2′ : Air is compressed from p_1 to p_x in the L.P. compressor.

Operation 2′-3 : Air is cooled in the intercooler at constant pressure p_x.

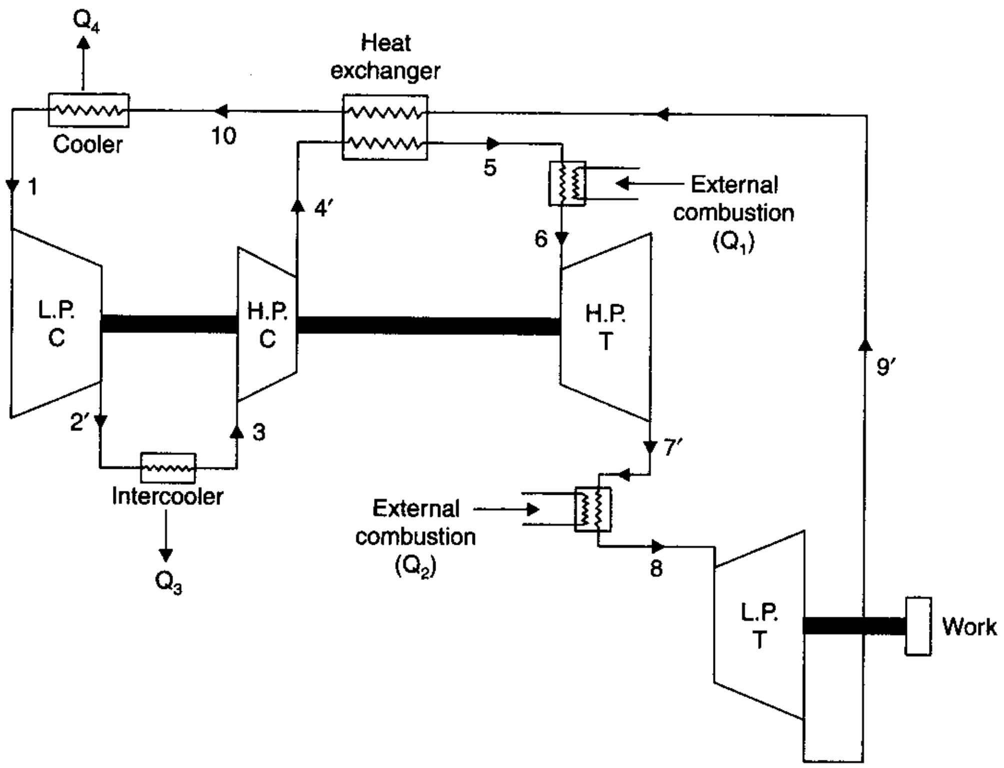


Fig. 13.50. Closed cycle gas turbine plant.

Operation 3-4′ : Air is compressed in the H.P. compressor from p_x to p_2.

Operation 4′-5 : High pressure air is heated at constant pressure by exhaust gases from power turbine in the heat exchanger to T_5.

Operation 5-6 : High pressure air further heated at constant pressure to the maximum temperature T_6 by an air heater (through external combustion).

Operation 6-7′ : The air is expanded in the H.P. turbine from p_2 to p_x producing work to drive the compressor.

Operation 7′-8 : Exhaust air from the H.P. turbine is heated at constant pressure in the air heater (through external combustion) to the maximum temperature $T_8 (= T_6)$.

Operation 8-9′ : The air is expanded in the L.P. turbine from p_x to p_1, producing energy for a flow of work externally.

Operation 9′-10 : Air from L.P. turbine is passed to the heat exchanger where energy is transferred to the air delivered from the H.P. compressor. The temperature of air leaving the heat exchanger and entering the cooler is T_{10}.

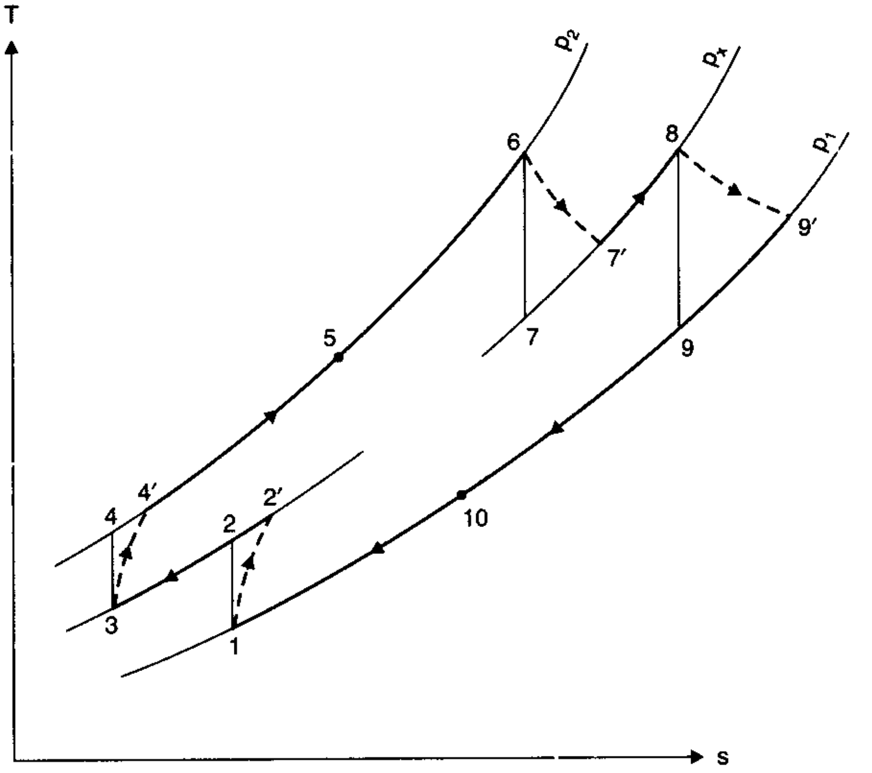


Fig. 13.51. *T-s* diagram for the plant.

Operation 10-11 : Air cooled to T_1 by the cooler before entering the L.P. compressor.

The energy balance for the whole plant is as follows :

$$Q_1 + Q_2 - Q_3 - Q_4 = W$$

In a closed cycle plant, in practice, the control of power output is achieved by varying the mass flow by the use of a reservoir in the circuit. The *reservoir maintains the design pressure and temperature and therefore achieves an approximately constant level of efficiency for varying* loads. In this cycle since it is closed, *gases other than air with favourable properties can be used* ; furthermore it is possible to burn solid fuels in the combustion heaters. *The major factor responsible for inefficiency in this cycle is the large irreversible temperature drop which occurs in the air heaters between the furnace and circulating gas.*

Note 1. In a closed cycle gas turbines, although air has been extensively used, the use of 'helium' which though of a lower density, has been inviting the attention of manufacturers for its use, for large output gas turbine units. *The specific heat of helium at constant pressure is about 'five times' that of air, therefore for each kg mass flow the heat drop and hence energy dealt with in helium machines is nearly five times of those in case of air*. The *surface area of the heat exchanger for helium can be kept as low as 1/3* of that *required for gas turbine plant using air as working medium*. For the same temperature ratio and for the plants of the same output the *cross-sectional area required for helium is much less than that for air*. It may therefore be concluded that the *size of helium unit is considerably small comparatively*.

2. Some gas turbine plants work on a combination of two cycles the open cycle and the closed cycle. Such a combination is called the *semi-closed cycle. Here a part of the working fluid is confined within the plant and another part flows from and to atmosphere.*

13.10.8. Gas Turbine Fuels

The various fuels used in gas turbines are enumerated and discussed below :

1. Gaseous fuels
2. Liquid fuels
3. Solid fuels

1. **Gaseous fuels.** *Natural gas is the ideal fuel for gas turbines*, but this is not available everywhere.

Blast furnace and *producer gases* may also be used for gas turbine power plants.

2. **Liquid fuels.** Liquid fuels of petroleum origin such as distillate oils or residual oils are most commonly used for gas turbine plant. The essential qualities of these fuels include *proper volatility, viscosity and calorific value*. At the same time it *should be* free *from any contents of moisture* and *suspended impurities that would log the small passages of the nozzles and damage valves and plungers of the fuel pumps.*

Minerals like *sodium, vanadium* and *calcium* prove *very harmful* for the *turbine blading* as these build deposits or corrode the blades. The sodium in ash should be less than 30% of the vanadium content as otherwise the ratio tends to be critical. The actual sodium content may be between 5 ppm to 10 ppm (part per million). If the vanadium is over 2 ppm, the magnesium in ash tends to become critical. *It is necessary that the magnesium in ash is at least three times the quantity of vanadium.* The content of calcium and lead should not be over 10 ppm and 5 ppm respectively.

Sodium is removed from residual oils by mixing with 5% of water and then double centrifuging when sodium leaves with water. Magnesium is added to the washed oil in the form of epsom salts, before the oil is sent into the combustor. This checks the corrosive action of vanadium. Residual oils burn with less ease than distillate oils and the latter are often used to start the unit from cold, after which the residual oils are fed in the combustor. In cold conditions residual oils need to be preheated.

3. **Solid fuels.** The use of solid fuels such as coal in pulverised form in gas turbines presents several difficulties most of which have been only partially overcome yet. The pulverising plant for coal in gas turbines applications is much lighter and small than its counterpart in steam generators. *Introduction of fuel in the combustion chamber of a gas turbine is required to be done against a high pressure whereas the pressure in the furnace of a steam plant is atmospheric.* Furthermore, the *degree of completeness of combustion in gas turbine applications has to be very high as otherwise soot and dust in gas would deposit on the turbine blading.*

Some practical applications of solid fuel burning in turbine combustors have been commercially, made available in recent years. In one such design finely crushed coal is used instead of pulverised fuel. This fuel is carried in stream of air tangentially into one end of a cylindrical furnace while gas comes out at the centre of opposite end. As the fuel particles roll around the circumference of the furnace they are burnt and a high temperature of about 1650°C is maintained which causes the mineral matter of fuel to be converted into a liquid slag. The slag covers the walls of the furnace and runs out through a top hole in the bottom. The result is that fly-ash is reduced to a very small content in the gases. In *another design* a regenerator is used to transfer the heat to air, the combustion chamber being located on the outlet of the turbine, and the combustion is carried out in the turbine exhaust stream. The advantage is that only clean air is handled by the turbine.

Example 13.32. *Air enters the compressor of a gas turbine plant operating on Brayton cycle at 101.325 kPa, 27°C. The pressure ratio in the cycle is 6. Calculate the maximum temperature in the cycle and the cycle efficiency. Assume $W_T = 2.5\ W_C$, where W_T and W_C are the turbine and the compressor work respectively. Take $\gamma = 1.4$.* **(P.U.)**

Solution. Pressure of intake air, p_1 = 101.325 kPa

Temperature of intake air, T_1 = 27 + 273 = 300 K

The pressure ratio in the cycle, r_p = 6

(*i*) **Maximum temperature in the cycle, T_3 :**

Refer Fig. 13.52.

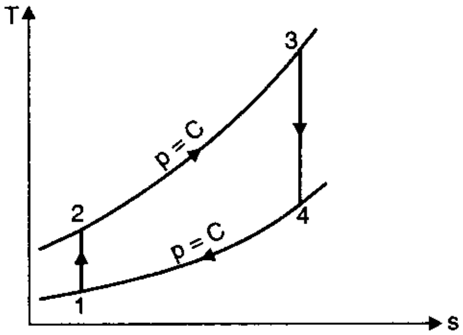


Fig. 13.52

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (r_p)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{1.4-1}{1.4}} = 1.668$$

$\therefore$ $$T_2 = 1.668\ T_1 = 1.668 \times 300 = 500.4 \text{ K}$$

Also, $$\frac{T_3}{T_4} = (r_p)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{1.4-1}{1.4}} = 1.668$$

$\therefore$ $$T_4 = \frac{T_3}{1.668}$$

But, $$W_T = 2.5\ W_C \quad \text{(given)}$$

$\therefore$ $$mc_p\,(T_3 - T_4) = 2.5\ mc_p\,(T_2 - T_1)$$

or, $$T_3 - \frac{T_3}{1.668} = 2.5\,(500.4 - 300) = 501 \quad \text{or} \quad T_3\left(1 - \frac{1}{1.668}\right) = 501$$

$\therefore$ $$T_3 = \frac{501}{\left(1 - \frac{1}{1.668}\right)} = \textbf{1251 K or 978°C.}\ \textbf{(Ans.)}$$

(*ii*) **Cycle efficiency, η_{cycle} :**

Now, $$T_4 = \frac{T_3}{1.668} = \frac{1251}{1.668} = 750 \text{ K}$$

$$\therefore \quad \eta_{cycle} = \frac{\text{Net work}}{\text{Heat added}} = \frac{mc_p(T_3 - T_4) - mc_p(T_2 - T_1)}{mc_p(T_3 - T_2)}$$

$$= \frac{(1251 - 750) - (500.4 - 300)}{(1251 - 500.4)} = \mathbf{0.4 \text{ or } 40\%.} \text{ (Ans.)}$$

$$\left[\textbf{Check}\,;\ \eta_{cycle} = 1 - \frac{1}{(r_p)^{\left(\frac{\gamma-1}{\gamma}\right)}} = 1 - \frac{1}{(6)^{\frac{1.4-1}{1.4}}} = \mathbf{0.4 \text{ or } 40\%.} \text{ (Ans.)} \right]$$

Example 13.33. *A gas turbine is supplied with gas at 5 bar and 1000 K and expands it adiabatically to 1 bar. The mean specific heat at constant pressure and constant volume are 1.0425 kJ/kg K and 0.7662 kJ/kg K respectively.*

(i) Draw the temperature-entropy diagram to represent the processes of the simple gas turbine system.

(ii) Calculate the power developed in kW per kg of gas per second and the exhaust gas temperature. **(GATE, 1995)**

Solution. *Given* : p_1 = 1 bar ; p_2 = 5 bar ; T_3 = 1000 K ; c_p = 1.0425 kJ/kg K ; c_v = 0.7662 kJ/kg K

$$\gamma = \frac{c_p}{c_v} = \frac{1.0425}{0.7662} = 1.36$$

(*i*) **Temperature-entropy (T-s) diagram :**

Temperature-entropy diagram representing the processes of the simple gas turbine system is shown in Fig. 13.53.

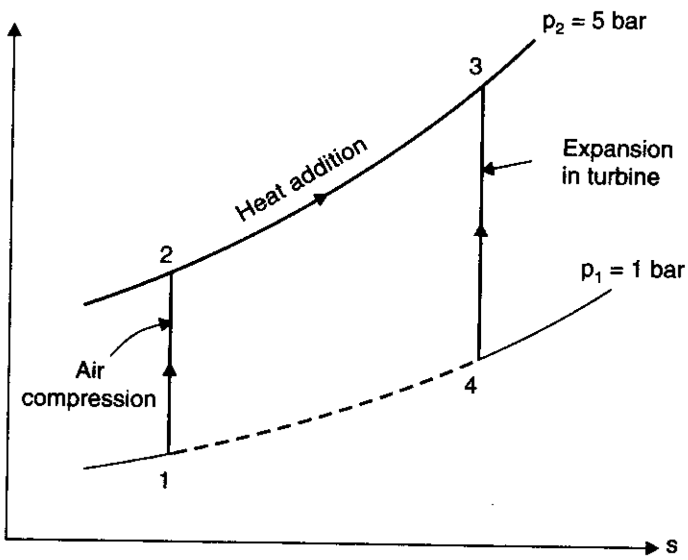


Fig. 13.53

(*ii*) **Power required :**

$$\frac{T_4}{T_3} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{5}\right)^{\frac{1.36-1}{1.36}} = 0.653$$

$\therefore \quad T_4 = 1000 \times 0.653 = 653$ K

Power developed per kg of gas per second

$$= c_p (T_3 - T_4)$$
$$= 1.0425 (1000 - 653) = \textbf{361.7 kW.} \quad \textbf{(Ans.)}$$

Example 13.34. *An isentropic air turbine is used to supply 0.1 kg/s of air at 0.1 MN/m² and at 285 K to a cabin. The pressure at inlet to the turbine is 0.4 MN/m². Determine the temperature at turbine inlet and the power developed by the turbine. Assume c_p= 1.0 kJ/kg K.*

(GATE, 1999)

Solution. *Given :* $\dot{m}_a$ = 0.1 kg/s ; p_1 = 0.1 MN/m² = 1 bar, T_4 = 285 K ; p_2 = 0.4 MN/m² = 4 bar ; c_p = 1.0 kJ/kg K.

Temperature at turbine inlet, T_3 :

$$\frac{T_3}{T_4} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}} = 1.486$$

$\therefore \quad T_3 = 285 \times 1.486 =$ **423.5 K. (Ans.)**

Power developed, P :

$$P = \dot{m}_a c_p (T_3 - T_4)$$
$$= 0.1 \times 1.0 (423.5 - 285)$$
$$= \textbf{13.85 kW.} \quad \textbf{(Ans.)}$$

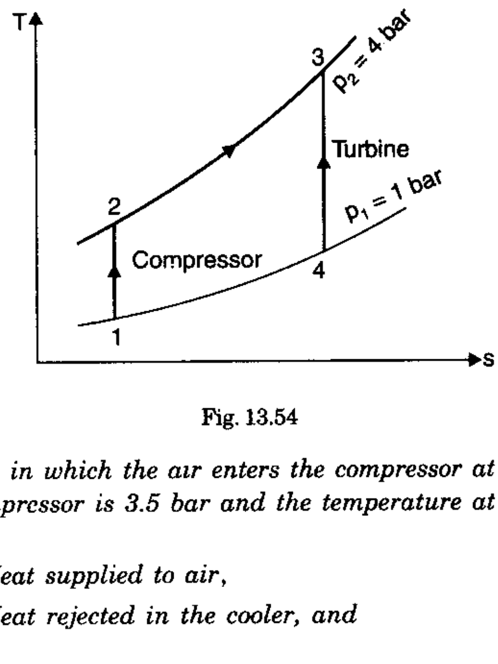


Fig. 13.54

Example 13.35. *Consider an air standard cycle in which the air enters the compressor at 1.0 bar and 20°C. The pressure of air leaving the compressor is 3.5 bar and the temperature at turbine inlet is 600°C. Determine per kg of air :*

(*i*) *Efficiency of the cycle,* (*ii*) *Heat supplied to air,*

(*iii*) *Work available at the shaft,* (*iv*) *Heat rejected in the cooler, and*

(*v*) *Temperature of air leaving the turbine.*

For air γ = 1.4 and c_p = 1.005 kJ/kg K.

Solution. Refer Fig. 13.52.

Pressure of air entering the compressor, p_1 = 1.0 bar

Temperature at the inlet of compressor, T_1 = 20 + 273 = 293 K

Pressure of air leaving the compressor, p_2 = 3.5 bar

Temperature of air at turbine inlet, T_3 = 600 + 273 = 873 K

(*i*) **Efficiency of the cycle, η_{cycle} :**

$$\eta_{cycle} = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{(3.5)^{\frac{1.4-1}{1.4}}} = \textbf{0.30 or 30\%.} \quad \textbf{(Ans.)} \quad \left(\because\ r_p = \frac{p_2}{p_1} = \frac{3.5}{1.0} = 3.5\right)$$

(*ii*) **Heat supplied to air :**

For compression process 1-2, we have

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{3.5}{1}\right)^{\frac{1.4-1}{1.4}} = 1.43$$

$\therefore$ $T_2 = T_1 \times 1.43 = 293 \times 1.43 \simeq 419$ K

$\therefore$ Heat supplied to air, $Q_1 = c_p (T_3 - T_2) = 1.005\ (873 - 419) =$ **456.27 kJ/kg. (Ans.)**

(*iii*) **Work available at the shaft, W :**

We know that, $\eta_{\text{cycle}} = \dfrac{\text{Work output }(W)}{\text{Heat input }(Q_1)}$

or $0.30 = \dfrac{W}{456.27}$ or $W = 0.3 \times 456.27 = 136.88$ kJ/kg

(*iv*) **Heat rejected in the cooler, Q_2 :**

Work output (W) = Heat supplied (Q_1) – heat rejected (Q_2)

$\therefore$ $Q_2 = Q_1 - W = 456.27 - 136.88 =$ **319.39 kJ/kg. (Ans.)**

(*v*) **Temperature of air leaving the turbine, T_4 :**

For *expansion (isentropic) process 3-4,* we have

$$\frac{T_3}{T_4} = (r_p)^{\frac{\gamma-1}{\gamma}} = (3.5)^{\frac{1.4-1}{1.4}} = 1.43$$

$$\therefore \quad T_4 = \frac{T_3}{1.43} = \frac{873}{1.43} = \textbf{610.5 K.} \quad \textbf{(Ans.)}$$

[**Check** : Heat rejected in the air cooler at constant pressure during the process 4-1 can also be calculated as : Heat rejected $= m \times c_p (T_4 - T_1) = 1 \times 1.005 \times (610.5 - 293) = 319.1$ kJ/kg]

Example 13.36. *A closed cycle ideal gas turbine plant operates between temperature limits of 800°C and 30°C and produces a power of 100 kW. The plant is designed such that there is no need for a regenerator. A fuel of calorific 45000 kJ/kg is used. Calculate the mass flow rate of air through the plant and rate of fuel consumption.*

Assume c_p = 1 kJ/kg K and γ = 1.4. **(GATE, 2000)**

Solution. *Given* : $T_1 = 30 + 273 = 303$ K ; $T_3 = 800 + 273 = 1073$ K ; $C = 45000$ kJ/kg ; c_p = 1 kJ/kg K ; $\gamma = 1.4$; $W_{\text{turbine}} - W_{\text{compressor}} = 100$ kW.

$\dot{m}_a$, $\dot{m}_f$:

Since no regenerator is used we can assume the turbine expands the gases upto T_4 in such a way that the exhaust gas temperature from the turbine is equal to the temperature of air coming out of the compressor *i.e.,* $T_2 = T_4$

$$\frac{p_2}{p_1} = \frac{p_3}{p_4},\ \frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \text{ and } \frac{p_3}{p_4} = \left(\frac{T_3}{T_4}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\therefore \quad \frac{T_2}{T_1} = \frac{T_3}{T_4} = \frac{T_3}{T_2}$$

$(\because\ T_2 = T_4$ ......assumed)

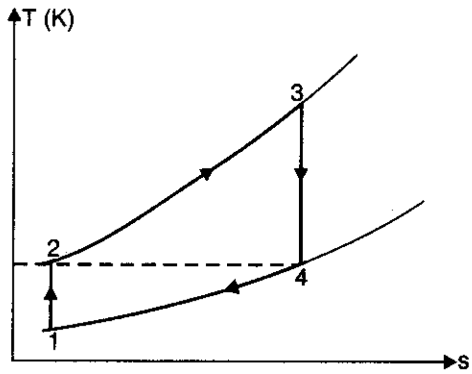


Fig. 13.55

or, $$T_2^2 = T_1T_3 \quad \text{or} \quad T_2 = \sqrt{T_1T_3}$$

or, $$T_2 = \sqrt{303 \times 1073} = 570.2 \text{ K}$$

Now, $$W_{\text{turbine}} - W_{\text{compressor}} = \dot{m}_f \times C \times \eta$$

or, $$100 = \dot{m}_f \times 45000 \times \left[1 - \frac{T_4 - T_1}{T_3 - T_2}\right]$$

$$= \dot{m}_f \times 45000 \left[1 - \frac{570.2 - 303}{1073 - 570.2}\right]$$

$$= \dot{m}_f \times 21085.9$$

or, $$\dot{\mathbf{m}}_{\mathbf{f}} = \frac{100}{21085.9} = \mathbf{4.74 \times 10^{-3}\ kg/s.}$$ **(Ans.)**

Again, $$W_{\text{turbine}} - W_{\text{compressor}} = 100 \text{ kW}$$

$$(\dot{m}_a + \dot{m}_f)(T_3 - T_4) - \dot{m}_a \times 1 \times (T_2 - T_1) = 100$$

or, $$(\dot{m}_a + 0.00474)(1073 - 570.2) - \dot{m}_a(570.2 - 303) = 100$$

or, $$(\dot{m}_a + 0.00474) \times 502.8 - 267.2\ \dot{m}_a = 100$$

or, $$502.8\ \dot{m}_a + 2.383 - 267.2\ \dot{m}_a = 100$$

or, $$235.6\ \dot{m}_a = 97.617$$

$\therefore$ $$\dot{\mathbf{m}}_{\mathbf{a}} = \mathbf{0.414\ kg/s.}$$ **(Ans.)**

Example 13.37. *In a gas turbine plant working on Brayton cycle, the air at inlet is 27°C, 0.1 MPa. The pressure ratio is 6.25 and the maximum temperature is 800°C. The turbine and compressor efficiencies are each 80%. Find compressor work, turbine work, heat supplied, cycle efficiency and turbine exhaust temperature. Mass of air may be considered as 1 kg. Draw T-s diagram.* **(N.U.)**

Solution. Refer to Fig. 13.56.

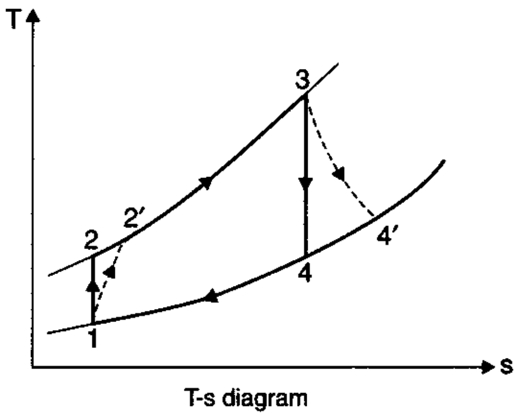


Fig. 13.56

Given : $T_1 = 27 + 273 = 300$ K ; $p_1 = 0.1$ MPa ; $r_p = 6.25$, $T_3 = 800 + 273 = 1073$ K ; $\eta_{\text{comp.}} = \eta_{\text{turbine}} = 0.8$.

For the *compression process 1-2*, we have

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (r_p)^{\frac{\gamma-1}{\gamma}} = (6.25)^{\frac{1.4-1}{1.4}} = 1.688$$

or $T_2 = 300 \times 1.688 = 506.4$ K

Also, $\eta_{comp.} = \frac{T_2 - T_1}{T_2' - T_1}$ or $0.8 = \frac{506.4 - 300}{T_2' - 300}$

or $T_2' = \frac{506.4 - 300}{0.8} + 300 = 558$ K

$\therefore$ **Compressor work,** $W_{comp.} = 1 \times c_p \times (T_2' - T_1)$

$= 1 \times 1.005 \, (558 - 300) =$ **259.29 kJ/kg. (Ans.)**

For *expansion process 3-4*, we have

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = (r_p)^{\frac{\gamma-1}{\gamma}} = (6.25)^{\frac{1.4-1}{1.4}} = 1.688$$

or $T_4 = \frac{T_3}{1.688} = \frac{1073}{1.688} = 635.66$ K

Also, $\eta_{turbine} = \frac{T_3 - T_4'}{T_3 - T_4}$ or $0.8 = \frac{1073 - T_4'}{1073 - 635.66}$

or $T_4' = 1073 - 0.8 \, (1073 - 635.66) = 723.13$ K

$\therefore$ **Turbine work,** $W_{turbine} = 1 \times c_p \times (T_3 - T_4')$ (neglecting fuel mass)

$= 1 \times 1.005 \, (1073 - 723.13) =$ **351.6 kJ/kg. (Ans.)**

Net work output, $W_{net} = W_{turbine} - W_{comp.} = 351.6 - 259.29 = 92.31$ kJ/kg

Heat supplied, $Q_s = 1 \times c_p \times (T_3 - T_2')$

$= 1 \times 1.005 \times (1073 - 558) =$ **517.57 kJ/kg. (Ans.)**

Cycle efficiency, $\eta_{cycle} = \frac{W_{net}}{Q_s} = \frac{92.31}{517.57} = 0.1783$ or **17.83%. (Ans.)**

Turbine exhaust temperature, $T_4' = 723.13$ K or **450.13°C. (Ans.)**

The *T-s* diagram is shown in Fig. 13.56.

Example 13.38. *Find the required air-fuel ratio in a gas turbine whose turbine and compressor efficiencies are 85% and 80%, respectively. Maximum cycle temperature is 875°C. The working fluid can be taken as air (c_p = 1.0 kJ/kg K, γ = 1.4) which enters the compressor at 1 bar and 27°C. The pressure ratio is 4. The fuel used has calorific value of 42000 kJ/kg. There is a loss of 10% of calorific value in the combustion chamber.* **(GATE, 1998)**

Solution. *Given :* $\eta_{turbine} = 85\%$; $\eta_{compressor} = 80\%$; $T_3 = 273 + 875 = 1148$ K ; $T_1 = 27 + 273 = 300$ K ; $c_p = 1.0$ kJ/kg K ; $\gamma = 1.4$; $p_1 = 1$ bar; $p_2 = 4$ bar (since pressure ratio is 4) ; $C = 42000$ kJ/kg K, $\eta_{cc} = 90\%$ (since loss in the combustion chamber is 10%)

For isentropic compression 1-2, we have

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (4)^{\frac{1.4-1}{1.4}} = 1.486$$

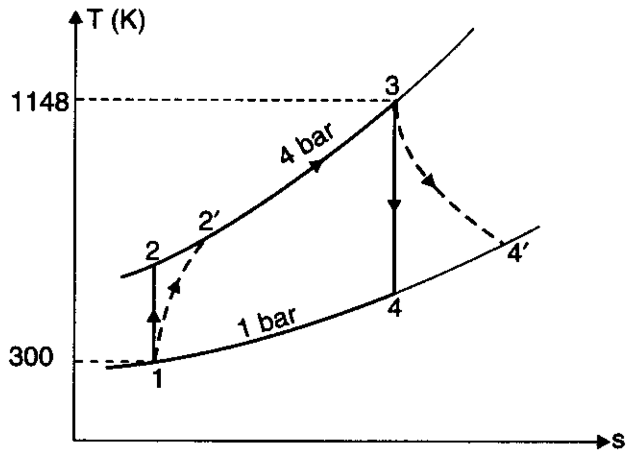


Fig. 13.57

$\therefore \quad T_2 = 300 \times 1.486 = 445.8 \text{ K}$

$$\eta_{\text{compressor}} = \frac{T_2 - T_1}{T_2' - T_1}$$

or $$0.8 = \frac{445.8 - 300}{T_2' - 300}$$

or $$T_2' = \frac{445.8 - 300}{0.8} + 300 = 482.2 \text{ K}$$

Now, heat supplied by the fuel = heat taken by the burning gases

$$0.9 \times m_f \times C = (m_a + m_f) \times c_p \times (T_3 - T_2')$$

$$\therefore \quad C = \left(\frac{m_a + m_f}{m_f}\right) \times \frac{c_p(T_3 - T_2')}{0.9} = \left(\frac{m_a}{m_f} + 1\right) \times \frac{c_p(T_3 - T_2')}{0.9}$$

or $$42000 = \left(\frac{m_a}{m_f} + 1\right) \times \frac{1.00(1148 - 482.27)}{0.9} = 739.78\left(\frac{m_a}{m_f} + 1\right)$$

$$\therefore \quad \frac{m_a}{m_f} = \frac{42000}{739.78} - 1 = 55.77 \text{ say } 56$$

$\therefore$ **A/F ratio = 56 : 1. (Ans.)**

Example 13.39. *A gas turbine unit receives air at 1 bar and 300 K and compresses it adiabatically to 6.2 bar. The compressor efficiency is 88%. The fuel has a heating valve of 44186 kJ/kg and the fuel-air ratio is 0.017 kJ/kg of air. The turbine internal efficiency is 90%. Calculate the work of turbine and compressor per kg of air compressed and thermal efficiency.*

For products of combustion, $c_p = 1.147$ *kJ/kg K and* $\gamma = 1.333$. **(UPSC, 1992)**

Solution. *Given :* $p_1 (= p_4) = 1$ bar, $T_1 = 300$ K ; $p_2 (= p_3) = 6.2$ bar ; $\eta_{\text{compressor}} = 88\%$;

$C = 44186$ kJ/kg ; Fuel-air ratio = 0.017 kJ/kg of air,

$\eta_{\text{turbine}} = 90\%$; $c_p = 1.147$ kJ/kg K ; $\gamma = 1.333$.

For isentropic compression process 1-2 :

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{6.2}{1}\right)^{\frac{1.4-1}{1.4}} = 1.684$$

$\therefore \quad T_2 = 300 \times 1.684 = 505.2$ K

Now, $\eta_{compressor} = \dfrac{T_2 - T_1}{T_2' - T_1}$

$$0.88 = \frac{505.2 - 300}{T_2' - 300}$$

$$T_2' = \left(\frac{505.2 - 300}{0.88} + 300\right)$$

$= 533.2$ K

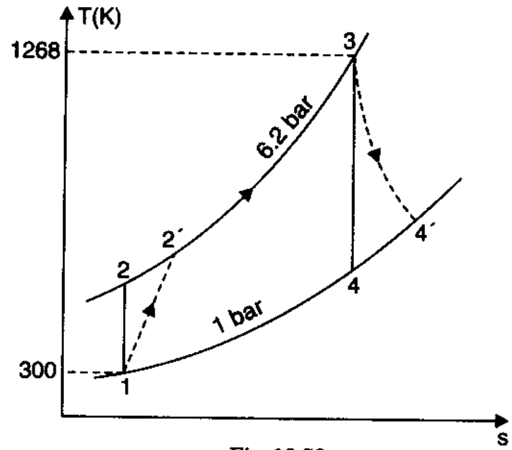


Fig. 13.58

Heat supplied $= (m_a + m_f) \times c_p(T_3 - T_2') = m_f \times C$

or $$\left(1 + \frac{m_f}{m_a}\right) \times c_p(T_3 - T_2') = \frac{m_f}{m_a} \times C$$

or $$(1 + 0.017) \times 1.005(T_3 - 533.2) = 0.017 \times 44186$$

$$\therefore \quad T_3 = \frac{0.017 \times 44186}{(1 + 0.017) \times 1.005} + 533.2 = 1268 \text{ K}$$

For isentropic expression process 3-4 :

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{6.2}\right)^{\frac{1.333-1}{1.333}} = 0.634$$

$\therefore \quad T_4 = 1268 \times 0.634 = 803.9$ K $\quad (\because \gamma_g = 1.333$ ......*Given*)

Now, $$\eta_{turbine} = \frac{T_3 - T_4'}{T_3 - T_4}$$

$$0.9 = \frac{1268 - T_4'}{1268 - 803.9}$$

$\therefore \quad T_4' = 1268 - 0.9(1268 - 803.9) = 850.3$ K

$W_{compressor} = c_p(T_2' - T_1) = 1.005(533.2 - 300) = 234.4$ kJ/kg

$W_{turbine} = c_{pg}(T_3 - T_4') = 1.147(1268 - 850.3) = 479.1$ kJ/kg

Net work $= W_{turbine} - W_{compressor}$

$= 479.1 - 234.4 = 244.7$ kJ/kg

Heat supplied per kg of air $= 0.017 \times 44186 = 751.2$ kJ/kg

$\therefore$ Thermal efficiency, $\eta_{th.} = \dfrac{\text{Net work}}{\text{Heat supplied}}$

$= \dfrac{244.7}{751.2} = 0.3257$ or **32.57%. (Ans.)**

Example 13.40. *The air enters the compressor of an open cycle constant pressure gas turbine at a pressure of 1 bar and temperature of 20°C. The pressure of the air after compression is 4 bar. The isentropic efficiencies of compressor and turbine are 80% and 85% respectively. The air-fuel ratio used is 90 : 1. If flow rate of air is 3.0 kg/s, find :*

(*i*) *Power developed.*

(*ii*) *Thermal efficiency of the cycle.*

Assume c_p *= 1.0 kJ/kg K and* γ *= 1.4 for air and gases.*

Calorific value of fuel = 41800 kJ/kg.

Solution. *Given* : p_1 = 1 bar ; T_1 = 20 + 273 = 293 K

p_2 = 4 bar ; $\eta_{compressor}$ = 80% ; $\eta_{turbine}$ = 85%

Air-fuel ratio = 90 : 1 ; Air flow rate, m_a = 3.0 kg/s

(*i*) **Power developed, P :**

Refer to Fig. 13.59 (*b*)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}} = 1.486$$

$$\therefore \quad T_2 = (20 + 273) \times 1.486 = 435.4 \text{ K}$$

$$\eta_{compressor} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.8 = \frac{435.4 - 293}{T_2' - 293}$$

$$\therefore \quad T_2' = \frac{435.4 - 293}{0.8} + 293 = 471 \text{ K}$$

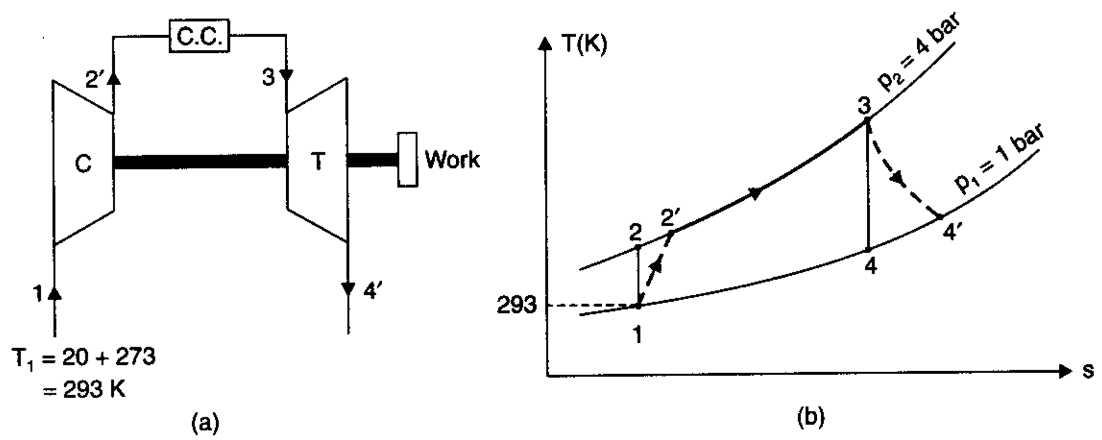


Fig. 13.59

Heat supplied by fuel = Heat taken by burning gases

$$m_f \times C = (m_a + m_f)\, c_p(T_3 - T_2')$$

(where m_a = mass of air, m_f = mass of fuel)

$\therefore \quad C = \left(\dfrac{m_a}{m_f} + 1\right) c_p (T_3 - T_2')$

$\therefore \quad 41800 = (90 + 1) \times 1.0 \times (T_3 - 471)$

i.e., $\quad T_3 = \dfrac{41800}{91} + 471 = 930 \text{ K}$

Again, $\quad \dfrac{T_4}{T_3} = \left(\dfrac{p_4}{p_3}\right)^{\frac{\gamma - 1}{\gamma}} = \left(\dfrac{1}{4}\right)^{\frac{0.4}{0.4}} = 0.672$

$\therefore \quad T_4 = 930 \times 0.672 = 624.9 \text{ K}$

$$\eta_{turbine} = \frac{T_3 - T_4'}{T_3 - T_4}$$

$$0.85 = \frac{930 - T_4'}{930 - 624.9}$$

$\therefore \quad T_4' = 930 - 0.85\ (930 - 624.9) = 670.6 \text{ K}$

$$W_{turbine} = m_g \times c_p \times (T_3 - T_4')$$

(where m_g is the mass of hot gases formed per kg of air)

$\therefore \quad W_{turbine} = \left(\dfrac{90 + 1}{90}\right) \times 1.0 \times (930 - 670.6)$

$= 262.28$ kJ/kg of air.

$W_{compressor} = m_a \times c_p \times (T_2' - T_1) = 1 \times 1.0 \times (471 - 293)$

$= 178$ kJ/kg of air

$W_{net} = W_{turbine} - W_{compressor}$

$= 262.28 - 178 = 84.28$ kJ/kg of air.

Hence *power developed,* $P = 84.28 \times 3 =$ **252.84 kW/kg of air. (Ans.)**

(*ii*) **Thermal efficiency of cycle, $\eta_{thermal}$:**

Heat supplied per kg of air passing through combustion chamber

$$= \frac{1}{90} \times 41800 = 464.44 \text{ kJ/kg of air}$$

$\therefore \quad \eta_{thermal} = \dfrac{\text{Work output}}{\text{Heat supplied}} = \dfrac{84.28}{464.44} = 0.1814$ or **18.14%. (Ans.)**

Example 13.41. *A gas turbine unit has a pressure ratio of 6 : 1 and maximum cycle temperature of 610°C. The isentropic efficiencies of the compressor and turbine are 0.80 and 0.82 respectively. Calculate the power output in kilowatts of an electric generator geared to the turbine when the air enters the compressor at 15°C at the rate of 16 kg/s.*

Take c_p = 1.005 kJ/kg K and γ = 1.4 for the compression process, and take c_p = 1.11 kJ/kg K and γ = 1.333 for the expansion process.

Solution. *Given* : $T_1 = 15 + 273 = 288$ K ; $T_3 = 610 + 273 = 883$ K ; $\dfrac{p_2}{p_1} = 6$,

$\eta_{compressor} = 0.80$; $\eta_{turbine} = 0.82$; Air flow rate = 16 kg/s

For compression process : c_p = 1.005 kJ/kg K, γ = 1.4

For expansion process : c_p = 1.11 kJ/kg K, γ = 1.333

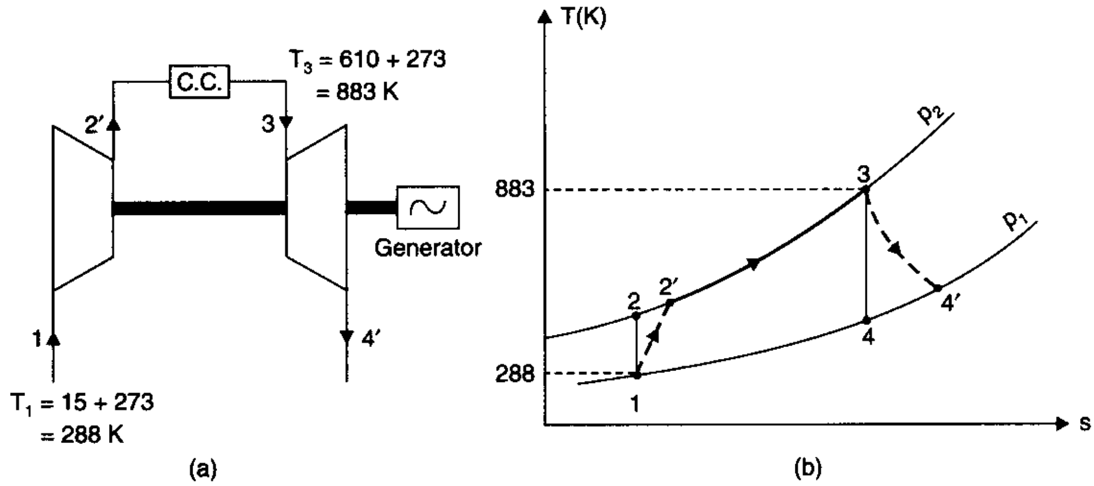


Fig. 13.60

In order to evaluate the net work output it is necessary to calculate temperatures T_2' and T_4'. To calculate these temperatures we must first calculate T_2 and then use the isentropic efficiency.

For an isentropic process, $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{1.4-1}{1.4}} = 1.67$$

$\therefore$ $$T_2 = 288 \times 1.67 = 481 \text{ K}$$

Also, $$\eta_{\text{compressor}} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.8 = \frac{481 - 288}{T_2' - T_1}$$

$\therefore$ $$T_2' = \frac{481 - 288}{0.8} + 288 = 529 \text{ K}$$

Similarly for the turbine, $$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{1.333-1}{1.333}} = 1.565$$

$\therefore$ $$T_4 = \frac{T_3}{1.565} = \frac{883}{1.565} = 564 \text{ K}$$

Also, $$\eta_{\text{turbine}} = \frac{T_3 - T_4'}{T_3 - T_4} = \frac{883 - T_4'}{883 - 564}$$

$$0.82 = \frac{883 - T_4'}{883 - 564}$$

$$T_4' = 883 - 0.82\,(883 - 564) = 621.4 \text{ K}$$

Hence,

Compressor work input, $W_{compressor} = c_p\ (T_2' - T_1)$
$= 1.005\ (529 - 288) = 242.2$ kJ/kg

Turbine work output, $W_{turbine} = c_p\ (T_3 - T_4')$
$= 1.11\ (883 - 621.4) = 290.4$ kJ/kg.

$\therefore$ Net work output, $W_{net} = W_{turbine} - W_{compressor}$
$= 290.4 - 242.2 = 48.2$ kJ/kg

Power in kilowatts $= 48.2 \times 16 =$ **771.2 kW. (Ans.)**

Example 13.42. *Calculate the thermal efficiency and work ratio of the plant is example 5.2, assuming that c_p for the combustion process is 1.11 kJ/kg K.*

Solution. Heat supplied $= c_p(T_3 - T_2')$
$= 1.11\ (883 - 529) = 392.9$ kJ/kg

$$\eta_{thermal} = \frac{\text{Net work output}}{\text{Heat supplied}} = \frac{48.2}{392.9} = 0.1226 \text{ or } \mathbf{12.26\%.}\ \ \textbf{(Ans.)}$$

Now, $$\textit{Work ratio} = \frac{\text{Net work output}}{\text{Gross work output}} = \frac{48.2}{W_{turbine}} = \frac{48.2}{290.4} = \mathbf{0.166.}\ \ \textbf{(Ans.)}$$

Example 13.43. *In a constant pressure open cycle gas turbine air enters at 1 bar and 20°C and leaves the compressor at 5 bar. Using the following data : Temperature of gases entering the turbine = 680°C, pressure loss in the combustion chamber = 0.1 bar, $\eta_{compressor}$ = 85%, $\eta_{turbine}$ = 80%, $\eta_{combustion}$ = 85%, γ = 1.4 and c_p = 1.024 kJ/kg K for air and gas, find :*

(i) The quantity of air circulation if the plant develops 1065 kW.

(ii) Heat supplied per kg of air circulation.

(iii) The thermal efficiency of the cycle.

Mass of the fuel may be neglected.

Solution. *Given* : $p_1 = 1$ bar, $p_2 = 5$ bar, $p_3 = 5 - 0.1 = 4.9$ bar, $p_4 = 1$ bar,
$T_1 = 20 + 273 = 293$ K, $T_3 = 680 + 273 = 953$ K,

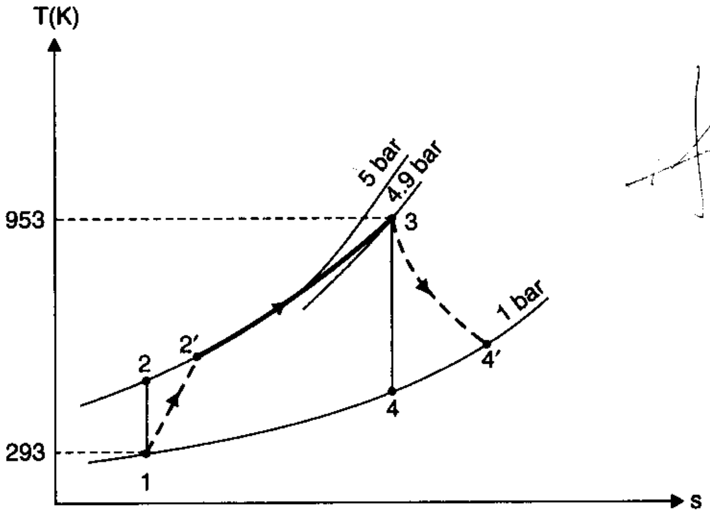


Fig. 13.61

$\eta_{compressor} = 85\%$, $\eta_{turbine} = 80\%$, $\eta_{combustion} = 85\%$.

For air and gases : $c_p = 1.024$ kJ/kg K, $\gamma = 1.4$

Power developed by the plant, $P = 1065$ kW.

(*i*) **The quantity of air circulation, m_a :**

For *isentropic compression 1-2,*

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}} = 1.584$$

$\therefore$ $$T_2 = 293 \times 1.584 = 464 \text{ K}$$

Now, $$\eta_{compressor} = \frac{T_2 - T_1}{T_2' - T_1} \quad i.e. \quad 0.85 = \frac{464 - 293}{T_2' - 293}$$

$\therefore$ $$T_2' = \frac{464 - 293}{0.85} + 293 = 494 \text{ K}$$

For *isentropic expansion process 3-4,*

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{4.9}\right)^{\frac{1.4-1}{1.4}} = 0.635$$

$\therefore$ $$T_4 = 953 \times 0.635 = 605 \text{ K}$$

Now, $$\eta_{turbine} = \frac{T_3 - T_4'}{T_3 - T_4}$$

$$0.8 = \frac{953 - T_4'}{953 - 605}$$

$\therefore$ $$T_4' = 953 - 0.8(953 - 605) = 674.6 \text{ K}$$

$$W_{compressor} = c_p\,(T_2' - T_1) = 1.024\,(494 - 293) = 205.8 \text{ kJ/kg}$$

$$W_{turbine} = c_p\,(T_3 - T_4') = 1.024\,(953 - 674.6) = 285.1 \text{ kJ/kg.}$$

$\therefore$ $$W_{net} = W_{turbine} - W_{compressor} = 285.1 - 205.8 = 79.3 \text{ kJ/kg of air}$$

If the mass of air flowing is m_a kg/s, the power developed by the plant is given by

$$P = m_a \times W_{net} \text{ kW}$$

$$1065 = m_a \times 79.3$$

$\therefore$ $$m_a = \frac{1065}{79.3} = 13.43 \text{ kg.}$$

i.e., **Quantity of air circulation = 13.43 kg. (Ans.)**

(*ii*) **Heat supplied per kg of air circulation :**

Actual heat supplied per kg of air circulation

$$= \frac{c_p\,(T_3 - T_2')}{\eta_{combustion}} = \frac{1.024\,(953 - 494)}{0.85} = 552.9 \text{ kJ/kg.}$$

(*iii*) **Thermal efficiency of the cycle, $\eta_{thermal}$:**

$$\eta_{thermal} = \frac{\text{Work output}}{\text{Heat supplied}} = \frac{79.3}{552.9} = 0.1434 \text{ or } \mathbf{14.34\%.} \textbf{ (Ans.)}$$

Example 13.44. *In a gas turbine the compressor is driven by the high pressure turbine. The exhaust from the high pressure turbine goes to a free low pressure turbine which runs the load. The air flow rate is 20 kg/s and the minimum and maximum temperatures are respectively 300 K and 1000 K. The compressor pressure ratio is 4. Calculate the pressure ratio of the low pressure turbine and the temperature of exhaust gases from the unit. The compressor and turbine are isentropic.* C_p *of air and exhaust gases = 1 kJ/kg K and* $\gamma = 1.4$. **(GATE, 1995)**

Solution. *Given :* $\dot{m}_a$ = 20 kg/s ; T_1 = 300 K ; T_3 = 1000 K, $\frac{p_2}{p_1} = 4$; c_p = 1 kJ/kg K ; γ = 1.4.

Pressure ratio of low pressure turbine, $\frac{p_4}{p_5}$ **:**

Since the compressor is driven by high pressure turbine,

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (4)^{\frac{0.4}{1.4}} = 1.486$$

or $\quad T_2 = 300 \times 1.486 = 445.8$ K

Also, $\quad \dot{m}_a c_p (T_2 - T_1) = \dot{m}_a c_p (T_3 - T_4)$

(neglecting mass of fuel)

or $\quad T_2 - T_1 = T_3 - T_4$

$445.8 - 300 = 1000 - T_4$, or $T_4 = 854.2$ K

For process 3-4 :

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{or} \quad \frac{p_3}{p_4} = \left(\frac{T_3}{T_4}\right)^{\frac{1.4}{0.4}}$$

or $$\frac{p_3}{p_4} = \left(\frac{1000}{854.2}\right)^{3.5} = 1.736$$

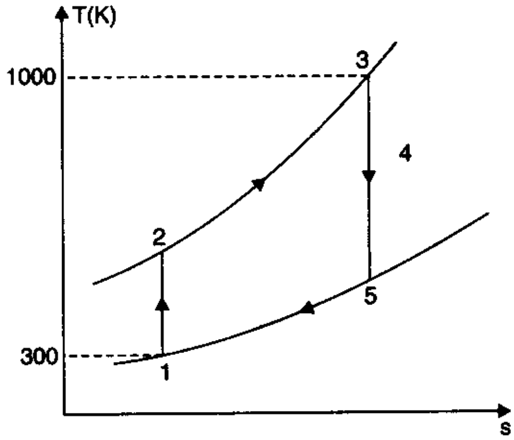


Fig. 13.62

Now, $$\frac{p_3}{p_4} = \frac{p_3}{p_5} \times \frac{p_5}{p_4} = 4 \times \frac{p_5}{p_4} \qquad \left(\because \frac{p_3}{p_5} = \frac{p_2}{p_1} = 4\right)$$

$$\therefore \quad \frac{p_5}{p_4} = \frac{1}{4}\left(\frac{p_3}{p_4}\right) = \frac{1}{4} \times 1.736 = 0.434$$

Hence pressure ratio of low pressure turbine = $\frac{p_4}{p_5} = \frac{1}{0.434}$ = **2.3. (Ans.)**

Temperature of the exhaust from the unit, $\mathbf{T_5}$ **:**

$$\frac{T_4}{T_5} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma-1}{\gamma}} = (2.3)^{\frac{1.4-1}{1.4}} = 1.269$$

$$\therefore \quad \mathbf{T_5} = \frac{T_4}{1.269} = \frac{854.2}{1.269} = \mathbf{673\ K.} \quad \textbf{(Ans.)}$$

☞**Example 13.45.** *Air is drawn in a gas turbine unit at 15°C and 1.01 bar and pressure ratio is 7 : 1. The compressor is driven by the H.P. turbine and L.P. turbine drives a separate* *power shaft. The isentropic efficiencies of compressor, and the H.P. and L.P. turbines are 0.82, 0.85 and 0.85 respectively. If the maximum cycle temperature is 610°C, calculate :*

(*i*) *The pressure and temperature of the gases entering the power turbine.*

(*ii*) *The net power developed by the unit per kg/s mass flow.*

(*iii*) *The work ratio.*

(*iv*) *The thermal efficiency of the unit.*

Neglect the mass of fuel and assume the following :

For compression process $c_{pa} = 1.005$ *kJ/kg K and* $\gamma = 1.4$

For combustion and expansion processes : $c_{pg} = 1.15$ *kJ/kg K and* $\gamma = 1.333$.

Solution. *Given* : $T_1 = 15 + 273 = 288$ K, $p_1 = 1.01$ bar, Pressure ratio $= \frac{p_2}{p_1} = 7$,

$\eta_{compressor} = 0.82$, $\eta_{turbine}$ (H.P.) $= 0.85$, $\eta_{turbine}$ (L.P.) $= 0.85$,

Maximum cycle temperature, $T_3 = 610 + 273 = 883$ K

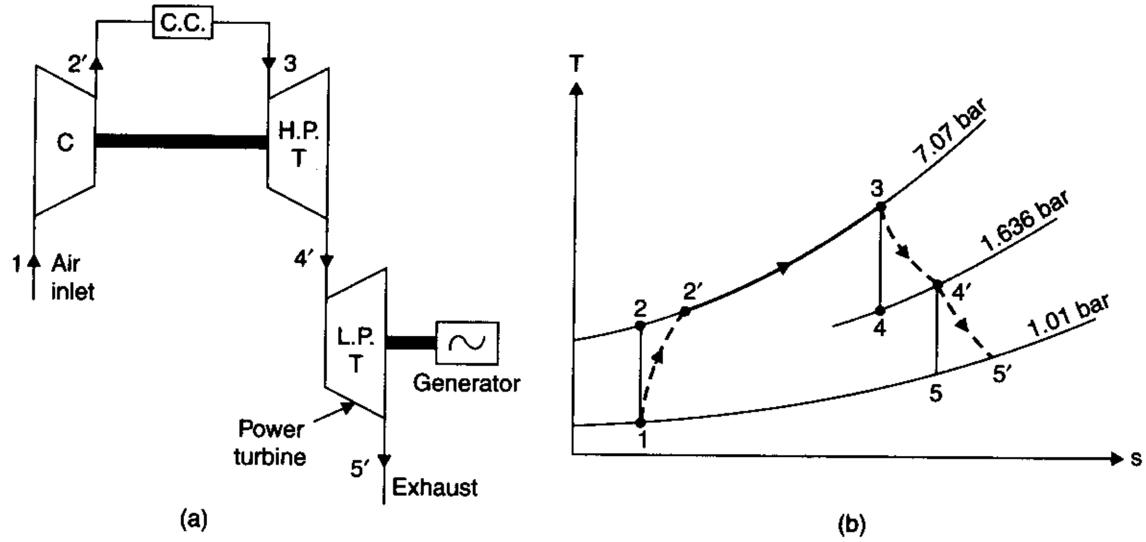


Fig. 13.63

(*i*) **Pressure and temperature of the gases entering the power turbine, p_4' and T_4' :**
Considering isentropic compression 1-2,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (7)^{\frac{1.4-1}{1.4}} = 1.745$$

$\therefore$ $$T_2 = 288 \times 1.745 = 502.5 \text{ K}$$

Also $$\eta_{compressor} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.82 = \frac{502.5 - 288}{T_2' - 288}$$

$$\therefore \quad T_2' = \frac{502.5 - 288}{0.82} + 288 = 549.6 \text{ K}$$

$$W_{\text{compressor}} = c_{pa}(T_2' - T_1) = 1.005 \times (549.6 - 288) = 262.9 \text{ kJ/kg}$$

Now, the *work output of H.P. turbine = Work input to compressor*

$$\therefore \quad c_{pg}(T_3 - T_4') = 262.9$$

i.e., $$1.15(883 - T_4') = 262.9$$

$$\therefore \quad T_4' = 883 - \frac{262.9}{1.15} = 654.4 \text{ K}$$

i.e., Temperature of gases entering the power turbine = **654.4 K. (Ans.)**

Again, for H.P. turbine :

$$\eta_{\text{turbine}} = \frac{T_3 - T_4'}{T_3 - T_4}$$

i.e., $$0.85 = \frac{883 - 654.4}{883 - T_4}$$

$$\therefore \quad T_4 = 883 - \left(\frac{883 - 654.4}{0.85}\right) = 614 \text{ K}$$

Now, considering isentropic expansion process 3-4,

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma - 1}{\gamma}}$$

or $$\frac{p_3}{p_4} = \left(\frac{T_3}{T_4}\right)^{\frac{\gamma}{\gamma - 1}} = \left(\frac{883}{614}\right)^{\frac{1.33}{0.33}} = 4.32$$

i.e., $$p_4 = \frac{p_3}{4.32} = \frac{7.07}{4.32} = 1.636 \text{ bar}$$

i.e., Pressure of gases entering the power turbine = **1.636 bar. (Ans.)**

(*ii*) **Net power developed per kg/s mass flow, P :**

To find the power output it is now necessary to calculate T_5'.

The pressure ratio, $\frac{p_4}{p_5}$, is given by $\frac{p_4}{p_3} \times \frac{p_3}{p_5}$

i.e., $$\frac{p_4}{p_5} = \frac{p_4}{p_3} \times \frac{p_2}{p_1} \qquad (\because \; p_2 = p_3 \text{ and } p_5 = p_1)$$

$$= \frac{7}{4.32} = 1.62$$

Then, $$\frac{T_4'}{T_5} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma - 1}{\gamma}} = (1.62)^{\frac{0.33}{1.33}} = 1.127$$

$$\therefore \quad T_5 = \frac{T_4'}{1.127} = \frac{654.4}{1.127} = 580.6 \text{ K.}$$

Again, for L.P. turbine :

$$\eta_{\text{turbine}} = \frac{T_4' - T_5'}{T_4' - T_5}$$

i.e.,

$$0.85 = \frac{654.4 - T_5'}{654.4 - 580.6}$$

$\therefore$

$$T_5' = 654.4 - 0.85\ (654.4 - 580.6) = 591.7\text{ K}$$

$$W_{\text{L.P. turbine}} = c_{pg}\ (T_4' - T_5') = 1.15\ (654.4 - 591.7) = 72.1\text{ kJ/kg}$$

Hence *net power output* (per kg/s mass flow)

$$= \mathbf{72.1\ kW.}\quad \textbf{(Ans.)}$$

(*iii*) **Work ratio :**

$$\textit{Work ratio} = \frac{\text{Net work output}}{\text{Gross work output}} = \frac{72.1}{72.1 + 262.9} = \mathbf{0.215.}\quad \textbf{(Ans.)}$$

(*iv*) **Thermal efficiency of the unit, η_{thermal} = ?**

Heat supplied $= c_{pg}(T_3 - T_2') = 1.15(883 - 549.6) = 383.4\text{ kJ/kg}$

$\therefore$

$$\eta_{\text{thermal}} = \frac{\text{Net work output}}{\text{Heat supplied}}$$

$$= \frac{72.1}{383.4} = 0.188 \text{ or } \mathbf{18.8\%.}\quad \textbf{(Ans.)}$$

Example 13.46. *In a gas turbine the compressor takes in air at a temperature of 15°C and compresses it to four times the initial pressure with an isentropic efficiency of 82%. The air is then passes through a heat exchanger heated by the turbine exhaust before reaching the combustion chamber. In the heat exchanger 78% of the available heat is given to the air. The maximum temperature after constant pressure combustion is 600°C, and the efficiency of the turbine is 70%. Neglecting all losses except those mentioned, and assuming the working fluid throughout the cycle to have the characteristic of air find the efficiency of the cycle.*

Assume R = 0.287 kJ/kg K and γ = 1.4 for air and constant specific heats throughout.

Solution. *Given* : $T_1 = 15 + 273 = 288$ K, Pressure ratio, $\frac{p_2}{p_1} = \frac{p_3}{p_4} = 4$, $\eta_{\text{compressor}} = 82\%$.

Effectiveness of the heat exchanger, $\varepsilon = 0.78$,

$\eta_{\text{turbine}} = 70\%$, Maximum temperature, $T_3 = 600 + 273 = 873$ K.

Efficiency of the cycle, η_{cycle} :

Considering the isentropic compression 1-2 :

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} = (4)^{\frac{1.4-1}{1.4}} = 1.486$$

$\therefore$

$$T_2 = 288 \times 1.486 = 428\text{ K}$$

Now,

$$\eta_{\text{compressor}} = \frac{T_2 - T_1}{T_2' - T_1}$$

i.e.,

$$0.82 = \frac{428 - 288}{T_2' - 288}$$

$$T_2' = \frac{428 - 288}{0.82} + 288 = 459\text{ K}$$

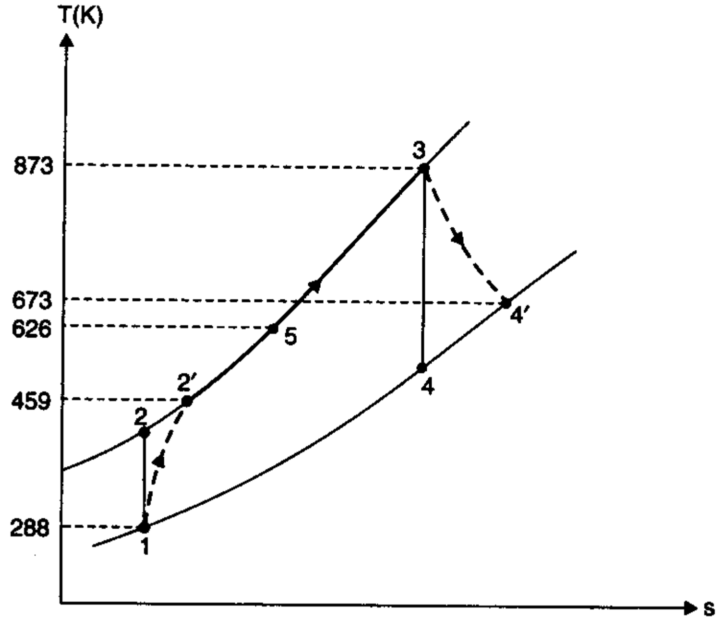


Fig. 13.64

Considering the isentropic expansion process 3-4 :

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = (4)^{\frac{1.4-1}{1.4}} = 1.486$$

$\therefore$ $$T_4 = \frac{T_3}{1.486} = \frac{873}{1.486} = 587.5 \text{ K.}$$

Again, $$\eta_{turbine} = \frac{T_3 - T_4'}{T_3 - T_4} = \frac{873 - T_4'}{873 - 587.5}$$

i.e., $$0.70 = \frac{873 - T_4'}{873 - 587.5}$$

$\therefore$ $$T_4' = 873 - 0.7(873 - 587.5) = 673 \text{ K}$$

$$W_{compressor} = c_p(T_2' - T_1)$$

But $$c_p = R \times \frac{\gamma}{\gamma - 1} = 0.287 \times \frac{1.4}{1.4 - 1} = 1.0045 \text{ kJ/kg K}$$

$\therefore$ $$W_{compressor} = 1.0045(459 - 288) = 171.7 \text{ kJ/kg}$$

$$W_{turbine} = c_p(T_3 - T_4') = 1.0045(873 - 673) = 200.9 \text{ kJ/kg}$$

$\therefore$ $$\text{Net work} = W_{turbine} - W_{compressor} = 200.9 - 171.7 = 29.2 \text{ kJ/kg.}$$

Effectiveness for heat exchanger, $\varepsilon = \dfrac{T_5 - T_2'}{T_4' - T_2'}$

i.e., $$0.78 = \frac{T_5 - 459}{673 - 459}$$

$\therefore$ $$T_5 = (673 - 459) \times 0.78 + 459 = 626 \text{ K}$$

∴ Heat supplied by fuel per kg

$$= c_p(T_3 - T_5) = 1.0045(873 - 626) = 248.1 \text{ kJ/kg}$$

∴ $$\eta_{cycle} = \frac{\text{Net work done}}{\text{Heat supplied by the fuel}} = \frac{29.2}{248.1}$$

$$= 0.117 \text{ or } \mathbf{11.7\%}. \quad \textbf{(Ans.)}$$

Example 13.47. *A gas turbine employs a heat exchanger with a thermal ratio of 72%. The turbine operates between the pressures of 1.01 bar and 4.04 bar and ambient temperature is 20°C. Isentropic efficiencies of compressor and turbine are 80% and 85% respectively. The pressure drop on each side of the heat exchanger is 0.05 bar and in the combustion chamber 0.14 bar. Assume combustion efficiency to be unity and calorific value of the fuel to be 41800 kJ/kg.*

Calculate the increase in efficiency due to heat exchanger over that for simple cycle.

Assume c_p is constant throughout and is equal to 1.024 kJ/kg K, and assume $\gamma = 1.4$.

For simple cycle the air-fuel ratio is 90 : 1, and for the heat exchange cycle the turbine entry temperature is the same as for a simple cycle.

Solution. Simple Cycle. Refer Fig. 13.65.

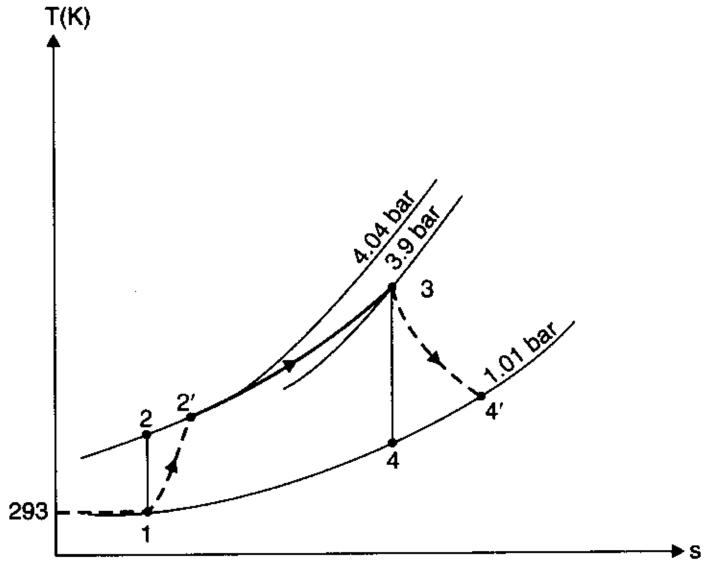


Fig. 13.65

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}} = 1.486$$

∴ $$T_2 = 293 \times 1.486 = 435.4$$

Also, $$\eta_{compressor} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.8 = \frac{435.4 - 293}{T_2' - 293}$$

$\therefore \quad T_2' = \dfrac{435.4 - 293}{0.8} + 293 = 471 \text{ K}$

Now $\quad m_f \times C = (m_a + m_f) \times c_p \times (T_3 - T_2')$

$[m_a = \text{mass of air}, \ m_f = \text{mass of fuel}]$

$\therefore \quad T_3 = \dfrac{m_f \times C}{c_p\,(m_a + m_f)} + T_2' = \dfrac{1 \times 41800}{1.024\,(90 + 1)} + 471 = 919.5 \text{ K}$

Also, $\quad \dfrac{T_4}{T_3} = \left(\dfrac{p_4}{p_3}\right)^{\frac{\gamma - 1}{\gamma}}$

or $\quad T_4 = T_3 \times \left(\dfrac{p_4}{p_3}\right)^{\frac{\gamma - 1}{\gamma}} = 919.5 \times \left(\dfrac{1.01}{3.9}\right)^{\frac{1.4 - 1}{1.4}} = 625 \text{ K}$

Again, $\quad \eta_{\text{turbine}} = \dfrac{T_3 - T_4'}{T_3 - T_4}$

$\therefore \quad 0.85 = \dfrac{919.5 - T_4'}{919.5 - 625}$

$\therefore \quad T_4' = 919.5 - 0.85(919.5 - 625) = 669 \text{ K}$

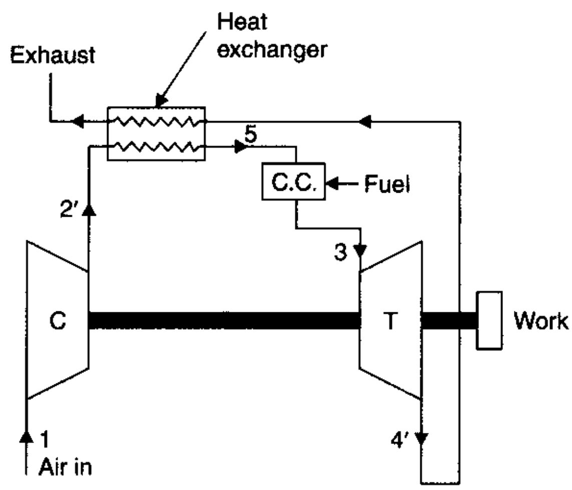
$$\eta_{\text{thermal}} = \frac{(T_3 - T_4') - (T_2' - T_1)}{(T_3 - T_2')}$$

$$= \frac{(919.5 - 669) - (471 - 293)}{(919.5 - 471)} = \frac{72.5}{448.5} = 0.1616 \text{ or } \mathbf{16.16\%}. \ \textbf{(Ans.)}$$

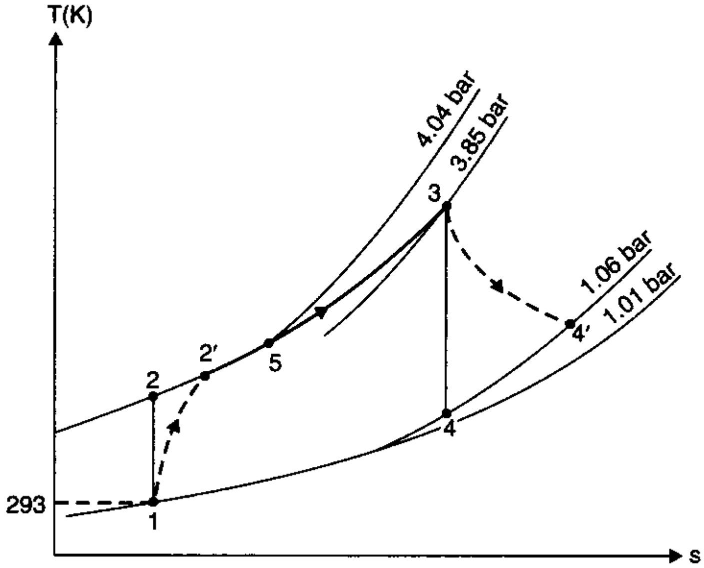
Heat Exchanger Cycle. Refer Figs. 13.66 (*a*) and (*b*)

$T_2' = 471$ K (as for simple cycle)

$T_3 = 919.5$ K (as for simple cycle)



(a)



(b)

Fig. 13.66

To find T_4' :

$$p_3 = 4.04 - 0.14 - 0.05 = 3.85 \text{ bar}$$

$$p_4 = 1.01 + 0.05 = 1.06 \text{ bar}$$

$$\therefore \quad \frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1.06}{3.85}\right)^{\frac{1.4-1}{1.4}} = 0.69$$

i.e., $$T_4 = 919.5 \times 0.69 = 634 \text{ K}$$

$$\eta_{turbine} = \frac{T_3 - T_4'}{T_3 - T_4} \; ; \; 0.85 = \frac{919.5 - T_4'}{919.5 - 634}$$

$$\therefore \quad T_4' = 919.5 - 0.85\,(919.5 - 634) = 677 \text{ K}$$

To find T_5 :

Thermal ratio (or effectiveness),

$$\varepsilon = \frac{T_5 - T_2'}{T_4' - T_2'} \qquad \therefore \quad 0.72 = \frac{T_5 - 471}{677 - 471}$$

$$\therefore \quad T_5 = 0.72(677 - 471) + 471 = 619 \text{ K}$$

$$\eta_{thermal} = \frac{(T_3 - T_4') - (T_2' - T_1)}{(T_3 - T_5)}$$

$$= \frac{(919.5 - 677) - (471 - 293)}{(919.5 - 619)} = \frac{64.5}{300.5} = 0.2146 \text{ or } \mathbf{21.46\%}$$

∴ **Increase in thermal efficiency** = 21.46 – 16.16 = **5.3%. (Ans.)**

☞**Example 13.48.** *A 5400 kW gas turbine generating set operates with two compressor stages, the overall pressure ratio is 9 : 1. A high pressure turbine is used to drive the compressors,* *and a low-pressure turbine drives the generator. The temperature of the gases at entry to the high pressure turbine is 625°C and the gases are reheated to 625°C after expansion in the first turbine. The exhaust gases leaving the low-pressure turbine are passed through a heat exchanger to heat the air leaving the high pressure stage compressor. The compressors have equal pressure ratios and intercooling is complete between the stages. The air inlet temperature to the unit is 20°C. The isentropic efficiency of each compressor stage is 0.8, and the isentropic efficiency of each turbine stage is 0.85, the heat exchanger thermal ratio is 0.8. A mechanical efficiency of 95% can be assumed for both the power shaft and compressor turbine shaft. Neglecting all pressure losses and changes in kinetic energy calculate :*

(i) The thermal efficiency *(ii) Work ratio of the plant*

(iii) The mass flow in kg/s.

Neglect the mass of the fuel and assume the following :

For air : c_{pa} *= 1.005 kJ/kg K and* $\gamma = 1.4$.

For gases in the combustion chamber and in turbines and heat exchanger, c_{pg} *= 1.15 kJ/kg K and* $\gamma = 1.333$.

Solution. Refer Fig. 13.67.

Given : $T_1 = 20 + 273 = 293$ K, $T_6 = T_8 = 625 + 273 = 898$ K

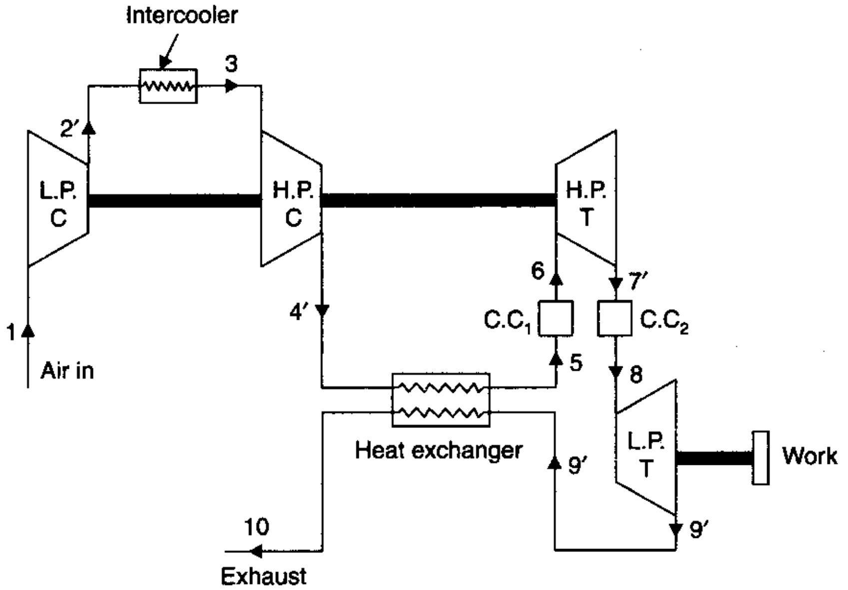
Efficiency of each compressor stage = 0.8

Efficiency of each turbine stage = 0.85

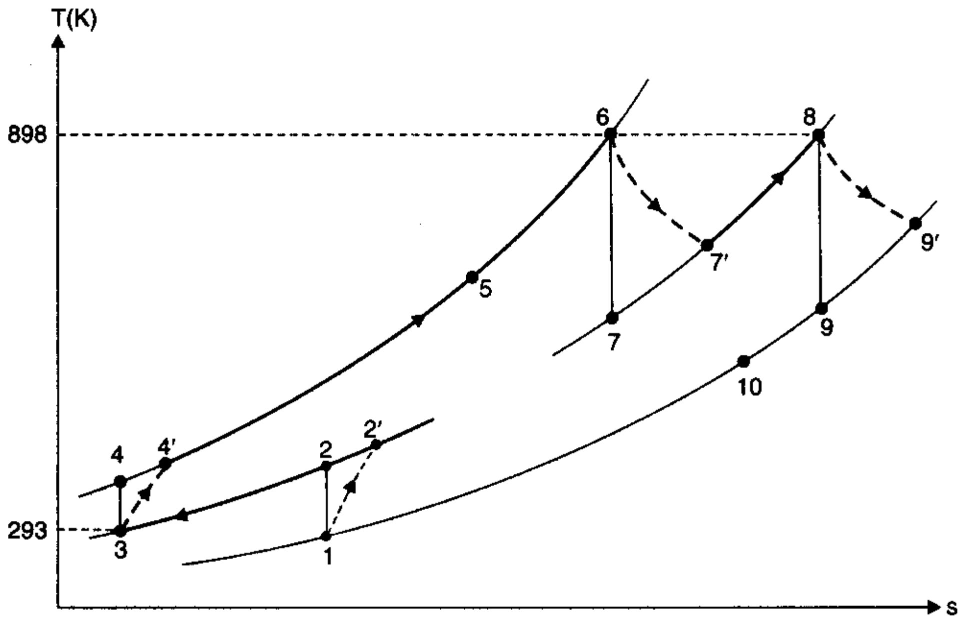
$\eta_{mech.} = 0.95$, $\varepsilon = 0.8$

(*i*) **Thermal efficiency,** $\eta_{thermal}$ **:**

Since the pressure ratio and the isentropic efficiency of each compressor is the same then the work input required for each compressor is the same since both compressor have the same air inlet temperature *i.e.*, $T_1 = T_3$ and $T_2' = T_4'$.



(a)



(b)

Fig. 13.67

Also, $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \text{ and } \frac{p_2}{p_1} = \sqrt{9} = 3$$

∴ $$T_2 = (20 + 273) \times (3)^{\frac{1.4-1}{1.4}} = 401 \text{ K}$$

Now, $$\eta_{\text{compressor}} \text{ (L.P.)} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.8 = \frac{401 - 293}{T_2' - 293}$$

i.e., $$T_2' = \frac{401 - 298}{0.8} + 293 = 428 \text{ K}$$

Work input per compressor stage

$$= c_{pa}(T_2' - T_1) = 1.005\ (428 - 293) = 135.6 \text{ kJ/kg}$$

The H.P. turbine is required to drive both compressors and to overcome mechanical friction.

i.e., Work output of H.P. turbine $= \dfrac{2 \times 135.6}{0.95} = 285.5$ kJ/kg

∴ $$c_{pg}\ (T_6 - T_7') = 285.5$$

i.e., $$1.15\ (898 - T_7') = 285.5$$

∴ $$T_7' = 898 - \frac{285.5}{1.15} = 650 \text{ K}$$

Now, $$\eta_{\text{turbine (H.P.)}} = \frac{T_6 - T_7'}{T_6 - T_7}\ ; \quad 0.85 = \frac{898 - 650}{898 - T_7}$$